MTH 1125 Test #1 - (12 pm class) - Solutions

Fall 2016

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2-8x+12}{x^2+4x-12} =$

Step #1 Try Plugging in:

$\lim_{x\to 2}\frac{x^2-8x+12}{x^2+4x-12} = \frac{(2)^2-8(2)+12}{(2)^2+4(2)-12} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 2}\frac{x^2-8x+12}{x^2+4x-12} = \lim_{x\to 2}\frac{(x-2)(x-6)}{(x+6)(x-2)} = \lim_{x\to 2}\frac{(x-6)}{(x+6)} = \frac{(2)-6}{(2)+6} = \frac{-4}{8} = -\frac{1}{2}$

i.e., $\lim_{x\to 2}\frac{x^2-8x+12}{x^2+4x-12} = -\frac{1}{2}$

2. Compute: $\lim_{x\to 3}\frac{x^2-4x-11}{x^2-6x+15} =$

Step #1 Try Plugging In:

$\lim_{x\to 3}\frac{x^2-4x-11}{x^2-6x+15} = \frac{(3)^2-4(3)-11}{(3)^2-6(3)+15} = -\frac{14}{6} = -\frac{7}{3}$

i.e., $\lim_{x\to 3}\frac{x^2-4x-11}{x^2-6x+15} = -\frac{7}{3}$

3. Compute: $\lim_{x\to 3} \frac{x^2+x-6}{x^2+7x-30} =$

Step #1 Try Plugging in:

$$\lim_{x\to 3} \frac{x^2+x-6}{x^2+7x-30} = \frac{(3)^2+(3)-6}{(3)^2+7(3)-30} = \frac{6}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to 3^-} \frac{x^2+x-6}{x^2+7x-30} = \lim_{x\to 3^-} \frac{x^2+x-6}{(x-3)(x+10)} = \frac{6}{(-\varepsilon)(13)} = \frac{\left(\frac{6}{13}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x - 3 < 0 \end{array}$$

$$\lim_{x\to 3^+} \frac{x^2+x-6}{x^2+7x-30} = \lim_{x\to 3^+} \frac{x^2+x-6}{(x-3)(x+10)} = \frac{6}{(+\varepsilon)(13)} = \frac{\left(\frac{6}{13}\right)}{(+\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x - 3 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to 3} \frac{x^2+x-6}{x^2+7x-30}$ **Does Not Exist!**

4. Compute: $\lim_{x\to -\infty} \frac{x^3+3x^2-8x}{9x^2+4x-5} =$

$$\lim_{x\to -\infty} \frac{x^3+3x^2-8x}{9x^2+4x-5} = \lim_{x\to -\infty} \frac{x^3}{9x^2} = \lim_{x\to -\infty} \frac{x}{9} = -\infty$$

i.e., $\lim_{x\to -\infty} \frac{x^3+3x^2-8x}{9x^2+4x-5} = -\infty$

5. $f(x) = \frac{x^2+x-6}{x^2-x-6}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = -2$ and $x = 3$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$\lim_{x\to -2^-} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to -2^-} \frac{x^2+x-6}{(x+2)(x-3)} = \frac{-4}{(-\varepsilon)(-5)} = \frac{\left(\frac{4}{5}\right)}{-\varepsilon} = -\infty$

$$\boxed{\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \end{array}}$$

$\lim_{x\to -2^+} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to -2^+} \frac{x^2+x-6}{(x+2)(x-3)} = \frac{-4}{(\varepsilon)(-5)} = \frac{\left(\frac{4}{5}\right)}{\varepsilon} = +\infty$

$$\boxed{\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$\lim_{x\to 3^-} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to 3^-} \frac{x^2+x-6}{(x+2)(x-3)} = \frac{6}{(5)(-\varepsilon)} = \frac{\left(\frac{6}{5}\right)}{(-\varepsilon)} = -\infty$

$$\boxed{\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \end{array}}$$

$\lim_{x\to 3^+} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to 3^+} \frac{x^2+x-6}{(x+2)(x-3)} = \frac{6}{(5)(\varepsilon)} = \frac{\left(\frac{6}{5}\right)}{(\varepsilon)} = +\infty$

$$\boxed{\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \end{array}}$$

Since the one-sided limits are **infinite,** $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

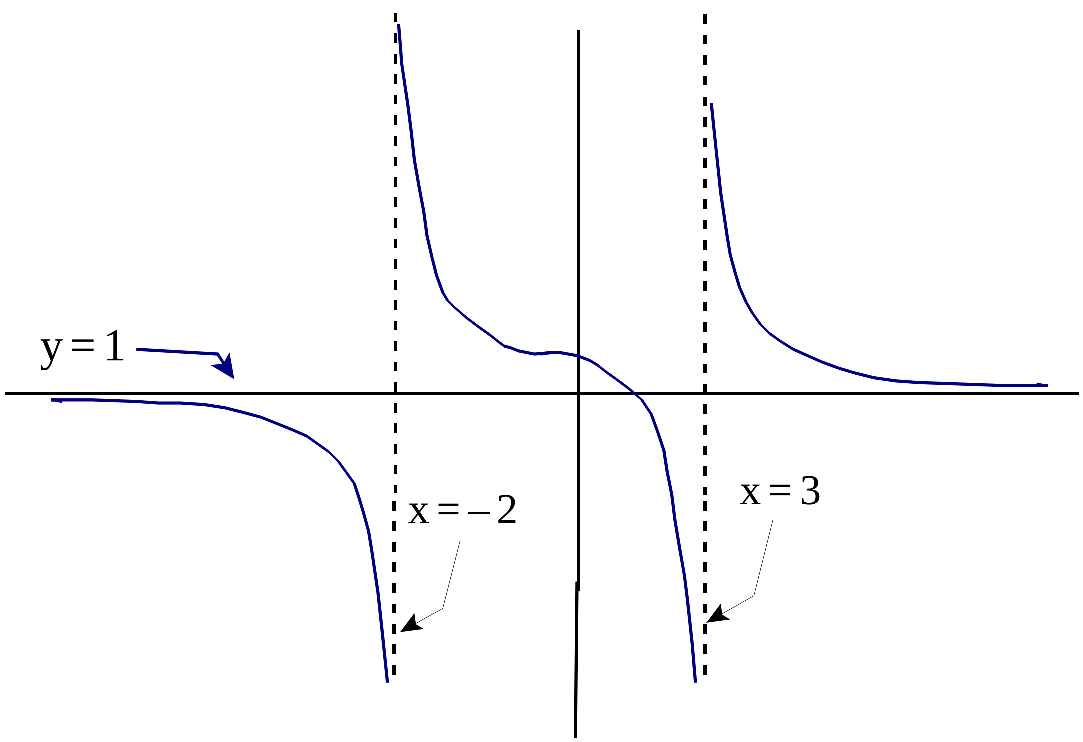
$\lim_{x\to+\infty} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptotes.*

Summary:

$\lim_{x\to-2^-} \frac{x^2+x-6}{x^2-x-6} = -\infty$	
$\lim_{x\to-2^+} \frac{x^2+x-6}{x^2-x-6} = +\infty$	$\lim_{x\to-\infty} \frac{x^2+x-6}{x^2-x-6} = 1$
$\lim_{x\to3^-} \frac{x^2+x-6}{x^2-x-6} = -\infty$	$\lim_{x\to+\infty} \frac{x^2+x-6}{x^2-x-6} = 1$
$\lim_{x\to3^+} \frac{x^2+x-6}{x^2-x-6} = +\infty$	

Graph $f(x) = \frac{x^2+x-6}{x^2-x-6}$



6. Compute: $\lim_{x\to 3} \frac{\sqrt{19-x}-4}{x-3} =$

Step #1 Try Plugging in:

$$\lim_{x\to 3} \frac{\sqrt{19-x}-4}{x-3} = \frac{\sqrt{19-(3)}-4}{(3)-3} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 3} \frac{\sqrt{19-x}-4}{x-3} = \lim_{x\to 3} \frac{\sqrt{19-x}-4}{x-3} \cdot \frac{\sqrt{19-x}+4}{\sqrt{19-x}+4} = \lim_{x\to 3} \frac{\left(\sqrt{19-x}\right)^2-(4)^2}{(x-3)\left[\sqrt{19-x}+4\right]}$$

$$= \lim_{x\to 3} \frac{(19-x)-16}{(x-3)\left[\sqrt{19-x}+4\right]} = \lim_{x\to 3} \frac{(3-x)}{(x-3)\left[\sqrt{19-x}+4\right]} = \lim_{x\to 3} \frac{-(x-3)}{(x-3)\left[\sqrt{19-x}+4\right]}$$

$$= \lim_{x\to 3} \frac{-1}{\left[\sqrt{19-x}+4\right]} = \lim_{x\to 3} \frac{-1}{\left[\sqrt{19-(3)}+4\right]} = \frac{-1}{[4+4]} = -\frac{1}{8}$$

i.e., $\lim_{x\to 3} \frac{\sqrt{19-x}-4}{x-3} = -\frac{1}{8}$

7.

$x =$	$f(x) =$
-2.5	-9.1
-2.1	-90.8
-2.01	-900.3
-2.001	$-9,000.3$
-2.0001	$-90,000.9$

$x =$	$f(x) =$
-1.5	-9.1
-1.9	-90.8
-1.99	-900.3
-1.999	$-9,000.3$
-1.9999	$-90,000.9$

Based on the information in the table above, do the following:

(a) $\lim_{x \to -2^-} f(x) = -\infty$

(b) $\lim_{x \to -2^+} f(x) = -\infty$

(c) Graph $f(x)$