MTH 1125 - Test 2 (9am Class) - Solutions

Fall 2017

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[5x^5+4x^4+3x^3+2x^2+x+4\sqrt{x}+2\right]=$

$\frac{d}{dx}\left[5x^5+4x^4+3x^3+2x^2+x+4x^{\frac{1}{2}}+2\right]$

$=5\left[5x^4\right]+4\left[4x^3\right]+3\left[3x^2\right]+2\left[2x\right]+1+2x^{-\frac{1}{2}}+0$

$=25x^4+16x^3+9x^2+4x+1+2x^{-\frac{1}{2}}$

i.e., $\frac{d}{dx}\left[5x^5+4x^4+3x^3+2x^2+x+4x^{\frac{1}{2}}+2\right]=25x^4+16x^3+9x^2+4x+1+2x^{-\frac{1}{2}}$

2. Compute: $\frac{d}{dx}\left[x^5\tan(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{x^5}_{1^{st}}\underbrace{\tan(x)}_{2^{nd}}\right]=\underbrace{5x^4}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\tan(x)}_{2^{nd}}+\underbrace{\sec^2(x)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{x^5}_{1^{st}}$$

$\frac{d}{dx}\left[x^5\tan(x)\right]=5x^4\tan(x)+\sec^2(x)\cdot x^5$

3. Compute: $\frac{d}{dx}\left[\frac{3x^2-6x+2}{\cos(x)}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{3x^2-6x+2}^{\text{top}}}{\underbrace{\cos(x)}_{\text{Bottom}}}\right]=\frac{\overbrace{(6x-6)}^{\text{top prime}}\cdot\overbrace{\cos(x)}^{\text{bottom}}-\overbrace{(-\sin(x))}^{\text{bottom prime}}\cdot\overbrace{\left(3x^2-6x+2\right)}^{\text{top}}}{\underbrace{(\cos(x))^2}_{\text{bottom squared}}}$$

i.e., $\frac{d}{dx}\left[\frac{3x^2-6x+2}{\cos(x)}\right]=\frac{(6x-6)\cos(x)+\sin(x)\left(3x^2-6x+2\right)}{\cos^2(x)}$

4. Compute: $\frac{d}{dx}\left[\left(6x^{20}+8x^{10}\right)^5\right] =$ This is the derivative of a function raised to a power.

$$\frac{d}{dx}\left[\left(6x^{20}+8x^{10}\right)^5\right] = \underbrace{5\left(6x^{20}+8x^{10}\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(120x^{19}+80x^9\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(6x^{20}+8x^{10}\right)^5\right] = 5\left(6x^{20}+8x^{10}\right)^4\left(120x^{19}+80x^9\right)$

5. Given that $f(x) = 2x^2 - 2x + 1$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2, 5)$.

We need two things:

i. A point on the line (We have that: $(x_1, y_1) = (2, 5)$)

ii. The slope of the line (This is $f'(x_1)$)

$f'(x) = 4x - 2$

At the point $(x_1, y_1) = (2, 5)$, **the slope is** $f'(2) = 4(2) - 2 = 6$

We will use the Point-Slope equation of a line:

$y - y_1 = m(x - x_1)$ (Where m is the slope and (x_1, y_1) is a known point on the line.)

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$y - 5 = 6(x - 2)$

The equation of the line tangent is $y - 5 = 6(x - 2)$

6. Given that $y = 3x^2 + 6x$ and that $x = \csc(t)$; compute $\frac{dy}{dt}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, *write explicitly the Liebniz form of the chain rule that you are going to use.)*

We know:

$\frac{dy}{dx} = 6x + 6$

$\frac{dx}{dt} = -\csc(t)\cot(t)$

We want: $\frac{dy}{dt}$

By the Liebniz form of the Chain Rule:

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = (6x+6)(-\csc(t)\cot(t)) = \underbrace{(6\csc(t)+6)(-\csc(t)\cot(t))}_{\substack{\text{express solely in terms of} \\ \text{independent variable } t}}$$

i.e. $\frac{dy}{dt} = (6\csc(t)+6)(-\csc(t)\cot(t))$

7. Compute: $\frac{d}{dx}\left[\sin\left(4x^2+8x+3\right)\right] =$

Outer:	$=$	$\sin(\quad)$
Deriv. of outer	$=$	$\cos(\quad)$

$$\frac{d}{dx}\left[\begin{array}{c} \sin\left(\underbrace{4x^2+8x+3}\right) \\ \uparrow \qquad\quad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{\cos\left(4x^2+8x+3\right)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{(8x+8)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

i.e., $\frac{d}{dx}\left[\sin\left(4x^2+8x+3\right)\right] = \cos\left(4x^2+8x+3\right)(8x+8)$

8. Compute: $\frac{d}{dx}\left[\left(\frac{2x^5+10x}{3x^3+9x}\right)^5\right] =$ In the broadest sense, this is the derivative of a *function raised to a power* - USE the CHAIN RULE.

$$\frac{d}{dx}\left[\underbrace{\left(\frac{2x^5+10x}{3x^3+9x}\right)^5}_{(g(x))^n}\right] = \underbrace{5\left(\frac{2x^5+10x}{3x^3+9x}\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\frac{2x^5+10x}{3x^3+9x}\right]\right)}_{\substack{\text{deriv of}\\ \text{inner Function}}}$$

$$= 5\left(\frac{2x^5+10x}{3x^3+9x}\right)^4 \underbrace{\frac{(10x^4+10)(3x^3+9x) - (9x^2+9)(2x^5+10x)}{(3x^3+9x)^2}}_{\substack{\text{quotient}\\ \text{rule}}}$$

i.e., $\frac{d}{dx}\left[\left(\frac{2x^5+10x}{3x^3+9x}\right)^5\right] = 5\left(\frac{2x^5+10x}{3x^3+9x}\right)^4 \cdot \frac{(10x^4+10)(3x^3+9x) - (9x^2+9)(2x^5+10x)}{(3x^3+9x)^2}$

9. Compute: $\frac{d}{dx}\left[\cot^5(4x^2+8x)\right] =$ Re-write!

$\frac{d}{dx}\left[(\cot(4x^2+8x))^5\right]$ This is the derivative of a function, raised to a power

$$\frac{d}{dx}\left[(\cot(4x^2+8x))^5\right] = \underbrace{5\left(\cot\left(4x^2+8x\right)\right)^4}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(\frac{d}{dx}\left[\cot\left(4x^2+8x\right)\right]\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

$$= 5(\cot(4x^2+8x))^4 \cdot \underbrace{\left(-\csc^2\left(4x^2+8x\right)\right)\cdot(8x+8)}_{\substack{\text{Chain}\\ \text{Rule}}}$$

i.e., $\frac{d}{dx}\left[\cot^5(4x^2+8x)\right] = 5(\cot(4x^2+8x))^4(-\csc^2(4x^2+8x))\cdot(8x+8)$

10. Given that $S'(x) = \frac{1}{2S(x)}$; compute $\frac{d}{dx}\left[S\left(x^2\right)\right]$

Outer:	$=$	$S(\quad)$
Deriv. of outer	$=$	$\frac{1}{2S(\quad)}$

$$\frac{d}{dx}\Big[\underset{\substack{\uparrow\\\text{outer}}}{S}\Big(\underset{\substack{\uparrow\\\text{inner}}}{\underbrace{x^2}}\Big)\Big] = \underbrace{\frac{1}{2S\left(x^2\right)}}_{\substack{\text{Deriv of outer,}\\\text{eval. at inner}}} \cdot \underbrace{2x}_{\substack{\text{deriv. of}\\\text{inner}}} = \frac{2x}{2S(x^2)} = \frac{x}{S(x^2)}$$

i.e., $\frac{d}{dx}\left[S\left(x^2\right)\right] = \frac{2x}{2S(x^2)} = \frac{x}{S(x^2)}$

11. Given that $f(x) = 4x^2 - 3x + 2$, compute $f'(x)$ **using the definition of derivative.** (i.e., using the "limit process.")

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[4(x+\Delta x)^2 - 3(x+\Delta x) + 2\right] - \left[4x^2 - 3x + 2\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[4\left(x^2 + 2x\Delta x + \Delta x^2\right) - 3(x+\Delta x) + 2\right] - \left[4x^2 - 3x + 2\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[4x^2 + 8x\Delta x + 4\Delta x^2 - 3x - 3\Delta x + 2\right] - \left[4x^2 - 3x + 2\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{8x\Delta x + 4\Delta x^2 - 3\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(8x + 4\Delta x - 3)}{\Delta x} = \lim_{\Delta x \to 0} (8x + 4\Delta x - 3) = 8x + 4(0) - 3 = 8x - 3$

i.e., $f'(x) = 8x - 3$