MTH 4441 Test #1

FALL 2021

Pat Rossi Name ______________________

1. **Define: Group**

2. **Define: Binary operation**

3. **Define:** Integers a and b **congruent modulo** n.

4. Give an alternate characterization of **congruence modulo** n.

5. **Define:** $(\mathbb{Z}_n, \oplus)$ (the **additive group of integers modulo** n)

6. **Define:** $(U_n, \odot)$ (the **multiplicative group of integers modulo** n)

7. **Prove:** If $(G, *)$ is a group, and a, b are any elements of G, then $(a * b)^{-1} = b^{-1} * a^{-1}$

8. **Define:** The **order of an element** x of a group $(G, *)$ (specify either **additive** or **multiplicative** notation.)

9. **Prove:** The identity element e in a group $(G, *)$ is unique.

10. Construct the group table for $(U_5, \odot)$

11. In the previous exercise, determine the order of the element 4

12. Construct the group table for $(\mathbb{Z}_4, \oplus)$

13. In the previous exercise, determine the order of the element 3

14. Determine whether the operation $*$, given by $a * b = \frac{a}{b^2+1}$ is a closed binary operation on the set $\mathbb{Z}$