MTH 1126 Test #3 - Solutions

Spring, 2005

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[\ln\left(\sin\left(x\right)\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(\sin\left(x\right)\right)\right]}_{\frac{d}{dx}[\ln(u)]} = \underbrace{\frac{1}{\sin\left(x\right)}}_{\frac{1}{u}} \cdot \underbrace{\cos\left(x\right)}_{\frac{du}{dx}} = \frac{\cos(x)}{\sin(x)} = \cot\left(x\right)$$

2. Compute: $\frac{d}{dx}\left[\ln\left(\frac{x^2+1}{3x^2-5x+4}\right)\right] =$

$$\underbrace{\frac{d}{dx}\left[\ln\left(\frac{x^2+1}{3x^2-5x+4}\right)\right]}_{\frac{d}{dx}[\ln(u)]} = \underbrace{\frac{1}{\left(\frac{x^2+1}{3x^2-5x+4}\right)}}_{\frac{1}{u}} \cdot \underbrace{\frac{\left(2x\right)\left(3x^2-5x+4\right)-\left(6x-5\right)\left(x^2+1\right)}{\left(3x^2-5x+4\right)^2}}_{\frac{du}{dx}}$$

$$= \frac{3x^2-5x+4}{x^2+1}\frac{(2x)\left(3x^2-5x+4\right)-(6x-5)\left(x^2+1\right)}{\left(3x^2-5x+4\right)^2} = \frac{(2x)\left(3x^2-5x+4\right)-(6x-5)\left(x^2+1\right)}{(x^2+1)(3x^2-5x+4)}$$

3. Compute: $\int \frac{x^2-2}{x^3-6x}dx =$

$$\int \frac{x^2-2}{x^3-6x}dx = \int \frac{1}{x^3-6x}\left(x^2-2\right)dx = \int \underbrace{\frac{1}{x^3-6x}}_{\frac{1}{u}}\underbrace{\left(x^2-2\right)dx}_{\frac{1}{3}du} = \int \frac{1}{u}\frac{1}{3}du = \frac{1}{3}\int \frac{1}{u}du$$

$$= \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln\left|x^3-6x\right| + C$$

	u	$=$	x^3-6x
$\Rightarrow$	$\frac{du}{dx}$	$=$	$3x^2-6$
$\Rightarrow$	du	$=$	$\left(3x^2-6\right)dx$
$\Rightarrow$	$\frac{1}{3}du$	$=$	$\left(x^2-2\right)dx$

4. Use the facts that $\ln\left(6\right) \approx 1.8$ and $\ln\left(5\right) \approx 1.6$ to approximate the following:

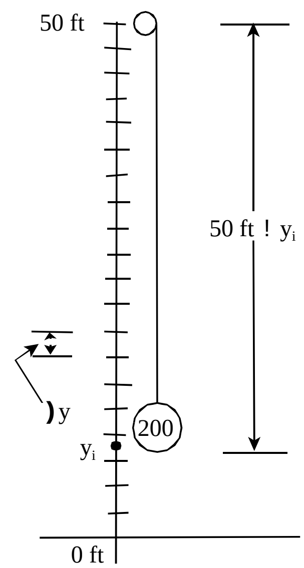
   (a) $\ln\left(30\right) = \ln\left(6\cdot 5\right) = \ln\left(6\right) + \ln\left(5\right) \approx 1.8 + 1.6$

   (b) $\ln\left(125\right) = \ln\left(5^3\right) = 3\ln\left(5\right) \approx 3\cdot\left(1.6\right) = 4.8$

   (c) $\ln\left(1.2\right) = \ln\left(\frac{6}{5}\right) = \ln\left(6\right) - \ln\left(5\right) \approx 1.8 - 1.6 = 0.2$

5. A crane is raising a wrecking ball from ground level to a height of 50 ft. The wrecking ball weighs 200 lb. and the cable weighs 2 lb per foot length. How much work is done in raising the wrecking ball?

   1. Draw a picture



   2. Partition the interval over which the work is done.

   3. Compute W_i, the work done in raising the ball from the bottom to the top of the i^{th} sub-interval.

   $W_i \approx F_i D_i$

$$\begin{aligned} F_i &= \text{(weight of unwound cable)} + \text{(weight of ball)} \\ &= \text{(length of unwound cable)}\,\text{(weight per unit length)} + \text{(weight of ball)} \\ &= (50\text{ ft } - y_i)\left(\tfrac{2\text{ lb}}{\text{ft}}\right) + (200\text{ lb}) \\ &= 100\text{ lb} - \tfrac{2\text{ lb}}{\text{ft}} y_i + (200\text{ lb}) \\ &= 300\text{ lb} - \tfrac{2\text{ lb}}{\text{ft}} y_i \end{aligned}$$

   i.e., $F_i = 300\text{ lb} - \frac{2\text{ lb}}{\text{ft}} y_i$

   Hence, $W_i \approx F_i D_i = \left(300\text{ lb } - \frac{2\text{ lb}}{\text{ft}} y_i\right) \Delta y$

   4. Approximate W_T, the total work done in raising the ball.

   $W_T \approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} \left(300\text{ lb } - \frac{2\text{ lb}}{\text{ft}} y_i\right) \Delta y$

   i.e., $W_T \approx \sum_{i=1}^{n} \left(300\text{ lb } - \frac{2\text{ lb}}{\text{ft}} y_i\right) \Delta y$

5. Let $\Delta y \to 0$

$W_T = \lim_{\Delta y \to 0} \sum_{i=1}^{n} \left(300 \text{ lb } - \frac{2 \text{ lb}}{\text{ft}} y_i\right) \Delta y = \int_{0 \text{ ft}}^{50 \text{ ft}} \left(300 \text{ lb } - \frac{2 \text{ lb}}{\text{ft}} y\right) dy$

$= \left[300 \text{ lb}\, y - \frac{2 \text{ lb}}{\text{ft}} \frac{y^2}{2}\right]_{0 \text{ ft}}^{50 \text{ ft}} = \left[300 \text{ lb}\, y - \frac{y^2 \text{ lb}}{\text{ft}}\right]_{0 \text{ ft}}^{50 \text{ ft}}$

$= \left(300 \text{ lb } (50 \text{ ft}) - \frac{(50 \text{ ft})^2 \text{ lb}}{\text{ft}}\right) - \left(300 \text{ lb } (0 \text{ ft}) - \frac{(0 \text{ ft})^2 \text{ lb}}{\text{ft}}\right)$

$= 15,000 \text{ lb ft} - 2,500 \text{ lb ft} = 12,500 \text{ lb ft}$