Mth 1125 Test #1

SUMMER 2004

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Instructions. Show clearly how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3} =$

   Step #1: Try plugging in: $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3} = \frac{3^2-9}{3^2-4(3)+3} = \frac{0}{0}$ no good - Zero divide

   Step #2: Cancel common factors, THEN plug in: $\lim_{x\to 3} \frac{x^2+x-2}{x^2-4x+3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{(x-1)(x-3)} = \lim_{x\to 3} \frac{(x+3)}{(x-1)} = \frac{(3+3)}{(3-1)} = 3$

   i.e., $\lim_{x\to 3} \frac{x^2-9}{x^2-4x+3} = 3$

2. Compute: $\lim_{x\to 2} \frac{2x^2-3x^2+7}{x^3-3} =$

   Step #1: Try plugging in: $\lim_{x\to 2} \frac{2x^2-3x^2+7}{x^3-3} = \frac{2(2)^2-3(2)^2+7}{(2)^3-3} = \frac{3}{5}$

   i.e., $\lim_{x\to 2} \frac{2x^2-3x^2+7}{x^3-3} = \frac{3}{5}$

3. Compute: $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} =$

   Step #1: Try plugging in: $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \frac{\sqrt{9-5}-2}{9-9} = \frac{0}{0}$ no good - Zero divide

   Step #2: Cancel common factors, THEN plug in:

   $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} \cdot \frac{\sqrt{x-5}+2}{\sqrt{x-5}+2} = \lim_{x\to 9} \frac{\left(\sqrt{x-5}\right)^2-2^2}{(x-9)\left(\sqrt{x-5}+2\right)} = \lim_{x\to 9} \frac{(x-5)-4}{(x-9)\left(\sqrt{x-5}+2\right)} = \lim_{x\to 9} \frac{(x-9)}{(x-9)\left(\sqrt{x-5}+2\right)}$

   $= \lim_{x\to 9} \frac{1}{\left(\sqrt{x-5}+2\right)} = \frac{1}{\left(\sqrt{9-5}+2\right)} = \frac{1}{4}$

   i.e., $\lim_{x\to 9} \frac{\sqrt{x-5}-2}{x-9} = \frac{1}{4}$

4. $\lim_{x\to\infty} \frac{4x^3-2x+5}{7x^3-5x^2+5x-2} =$

   As $x \to \infty$, terms of highest degree in numerator and denominator dominate the other terms> So:

   $\lim_{x\to\infty} \frac{4x^3-2x+5}{7x^3-5x^2+5x-2} = \lim_{x\to\infty} \frac{4x^3}{7x^3} = \lim_{x\to\infty} \frac{4}{7} = \frac{4}{7}$

   i.e., $\lim_{x\to\infty} \frac{4x^3-2x+5}{7x^3-5x^2+5x-2} = \frac{4}{7}$

5. Find asymptotes and graph: $f(x) = \frac{5x+3}{x-5}$

Verticals: find x-values that cause division by zero:

$x = 5$ causes division by zero.

$\lim_{x \to 5^-} \frac{5x+3}{x-5} = \frac{28}{-\varepsilon} = -\infty$

$$\begin{aligned} & x \to 5^- \\ \Rightarrow \quad & x < 5 \\ \Rightarrow \quad & x - 5 < 0 \end{aligned}$$

$\lim_{x \to 5^+} \frac{5x+3}{x-5} = \frac{28}{\varepsilon} = +\infty$

$$\begin{aligned} & x \to 5^+ \\ \Rightarrow \quad & x > 5 \\ \Rightarrow \quad & x - 5 > 0 \end{aligned}$$

Infinite limits tell us that $x = 5$ IS a vertical asymptote.

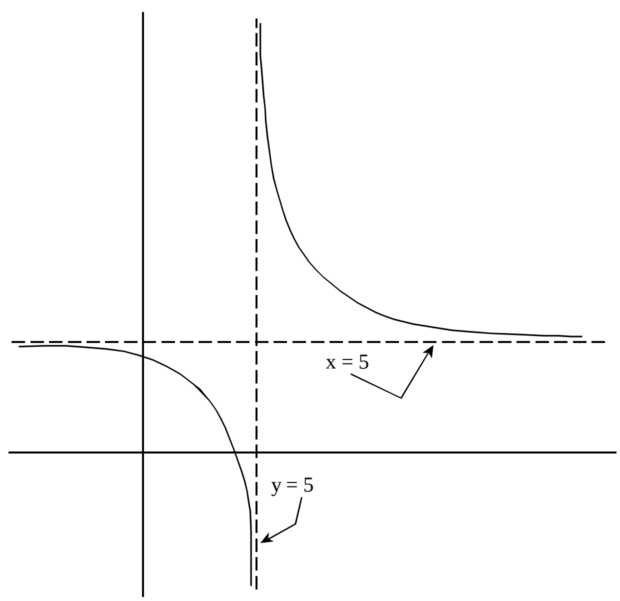
Horizontals: Let $x \to -\infty$ and $x \to \infty$

$\lim_{x \to -\infty} \frac{5x+3}{x-5} = \lim_{x \to -\infty} \frac{5x}{x} = \lim_{x \to -\infty} 5 = 5$

$\lim_{x \to \infty} \frac{5x+3}{x-5} = \lim_{x \to \infty} \frac{5x}{x} = \lim_{x \to \infty} 5 = 5$

Constant limits tell us that $y = 5$ is a horizontal asymptote.

Graph: $f(x) = \frac{5x+3}{x-5}$



6. $\lim_{x\to 3} \frac{x^2+3}{x^2-9} =$

Step #1: Try plugging in: $\lim_{x\to 3} \frac{x^2+3}{x^2-9} = \frac{12}{0}$ zero divide - no good

Step #2: this won't work for limits of the form: $\frac{\text{non-zero}}{\text{zero}}$

Step #3: Consider the one-sided limits:

$\lim_{x\to 3^-} \frac{x^2+3}{x^2-9} = \lim_{x\to 3^-} \frac{x^2+3}{(x-3)(x+3)} = \frac{12}{(-\varepsilon)(6)} = \frac{2}{-\varepsilon} = -\infty$

$$\begin{aligned} & x \to 3^- \\ \Rightarrow\ & x < 3 \\ \Rightarrow\ & x - 3 < 0 \end{aligned}$$

$\lim_{x\to 5^+} \frac{5x+3}{x-5} = \frac{28}{\varepsilon} = +\infty$

$$\begin{aligned} & x \to 3^+ \\ \Rightarrow\ & x > 3 \\ \Rightarrow\ & x - 3 > 0 \end{aligned}$$

$\lim_{x\to 3^+} \frac{x^2+3}{x^2-9} = \lim_{x\to 3^+} \frac{x^2+3}{(x-3)(x+3)} = \frac{12}{(\varepsilon)(6)} = \frac{2}{\varepsilon} = +\infty$

Since the one-sided limits are not equal, $\lim_{x\to 3} \frac{x^2+3}{x^2-9}$ Does Not Exist.

7. Given:

$x =$	$f(x)$
1.000	-3.1
2.000	-45.3
2.500	-559.2
2.900	-8547.3
2.990	-99131.8

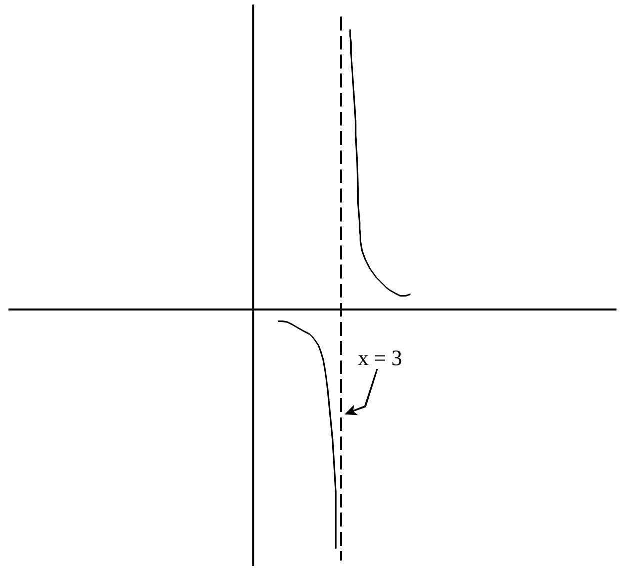
$x =$	$f(x)$
5.000	3.5
4.000	85.1
3.500	759.2
3.100	8412.7
3.010	95927.2

determine:

(a) $\lim_{x\to 3^-} f(x) = -\infty$

(b) $\lim_{x\to 3^+} f(x) = +\infty$

(c) Sketch a rough graph of $f(x)$.


x = 3