# MTH 1125 Test #1 - (2 pm class - Pod A) - Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2+x+8}{x^2+3x-8} =$

Step #1 Try Plugging In:

$\lim_{x\to 3} \frac{x^2+x+8}{x^2+3x-8} = \frac{(3)^2+(3)+8}{(3)^2+3(3)-8} = \frac{20}{10} = 2$

i.e., $\lim_{x\to 3} \frac{x^2+x+8}{x^2+3x-8} = 2$

2. Compute: $\lim_{x\to 4} \frac{x^2-9x+20}{x^2-5x+4} =$

$\lim_{x\to 4} \frac{x^2-9x+20}{x^2-5x+4} = \frac{(4)^2-9(4)+20}{(4)^2-5(4)+4} = \frac{0}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$\lim_{x\to 4} \frac{x^2-9x+20}{x^2-5x+4} = \lim_{x\to 4} \frac{(x-4)(x-5)}{(x-4)(x-1)} = \lim_{x\to 4} \frac{(x-5)}{(x-1)} = \frac{(4)-5}{(4)-1} = \frac{-1}{3} = -\frac{1}{3}$

i.e., $\lim_{x\to 4} \frac{x^2-9x+20}{x^2-5x+4} = -\frac{1}{3}$

3. Compute: $\lim_{x\to\ 4} \frac{x^2-4x-9}{x^2-2x-8} =$

Step #1 Try Plugging in:

$\lim_{x\to\ 4} \frac{x^2-4x-9}{x^2-2x-8} = \frac{(4)^2-4(4)-9}{(4)^2-2(4)-8} = \frac{-9}{0}$ No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good! "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to\ 4^-} \frac{x^2-4x-9}{x^2-2x-8} = \lim_{x\to\ 4^-} \frac{x^2-4x-9}{(x+2)(x-4)} = \frac{-9}{(6)(-\varepsilon)} = \frac{\left(-\frac{3}{2}\right)}{(-\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \end{array}$$

$$\lim_{x\to\ 4^+} \frac{x^2-4x-9}{x^2-2x-8} = \lim_{x\to\ 4^+} \frac{x^2-4x-9}{(x+2)(x-4)} = \frac{-9}{(6)(+\varepsilon)} = \frac{\left(-\frac{3}{2}\right)}{(+\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to\ 4} \frac{x^2-4x-9}{x^2-2x-8}$ **Does Not Exist!**

4. Compute: $\lim_{x\to\ -\infty} \frac{3x^4+4x^2-2}{2x^3+7x^2-x} =$

$$\lim_{x\to\ -\infty} \frac{3x^4+4x^2-2}{2x^3+7x^2-x} = \lim_{x\to\ -\infty} \frac{3x^4}{2x^3} = \lim_{x\to\ -\infty} \frac{3}{2}x = -\infty$$

i.e., $\lim_{x\to\ -\infty} \frac{3x^4+4x^2-2}{2x^3+7x^2-x} = -\infty$

5. $f(x) = \frac{x^2+5x}{x^2+x-6}$ Find the asymptotes and graph

Verticals

1. Find $x$-values that cause division by zero.

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) = 0$

$\Rightarrow x = -3$ and $x = 2$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to\ -3^-} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to\ -3^-} \frac{x^2+5x}{(x+3)(x-2)} = \frac{-6}{(-\varepsilon)(-5)} = \frac{-6}{(\varepsilon)(5)} = \frac{\left(-\frac{6}{5}\right)}{\varepsilon} = -\infty$$

$$\begin{array}{|ll|} & x \to -3^- \\ \Rightarrow & x < -3 \\ \Rightarrow & x+3 < 0 \end{array}$$

$$\lim_{x\to\ -3^+} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to\ -3^+} \frac{x^2+5x}{(x+3)(x-2)} = \frac{-6}{(+\varepsilon)(-5)} = \frac{\left(\frac{-6}{-5}\right)}{\varepsilon} = \frac{\left(\frac{6}{5}\right)}{\varepsilon} = +\infty$$

$$\begin{array}{|ll|} & x \to -3^+ \\ \Rightarrow & x > -3 \\ \Rightarrow & x+3 > 0 \end{array}$$

Since the one-sided limits are infinite, $x = -3$ *is a vertical asymptote.*

$$\lim_{x\to\ 2^-} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to\ 2^-} \frac{x^2+5x}{(x+3)(x-2)} = \frac{14}{(5)(-\varepsilon)} = \frac{\left(\frac{14}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\begin{array}{|ll|} & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x-2 < 0 \end{array}$$

$$\lim_{x\to 2^+} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to 2^+} \frac{x^2+5x}{(x+3)(x-2)} = \frac{14}{(5)(+\varepsilon)} = \frac{\left(\frac{14}{5}\right)}{(\varepsilon)} = +\infty$$

$$\begin{array}{|ll|} & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x-2 > 0 \end{array}$$

Since the one-sided limits are **infinite,** $x = 2$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

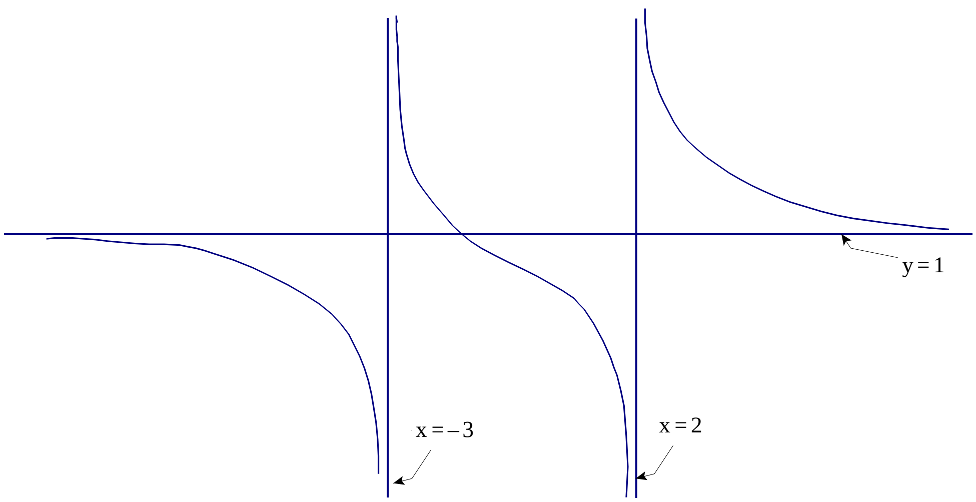
$\lim_{x\to+\infty} \frac{x^2+5x}{x^2+x-6} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are **finite** and **constant,** $y = 1$ *is a horizontal asymptote.*

Summary:

| | |
|---|---|
| $\lim_{x\to-3^-} \frac{x^2+5x}{x^2+x-6} = -\infty$ | |
| $\lim_{x\to-3^+} \frac{x^2+5x}{x^2+x-6} = +\infty$ | $\lim_{x\to-\infty} \frac{x^2+5x}{x^2+x-6} = 1$ |
| $\lim_{x\to2^-} \frac{x^2+5x}{x^2+x-6} = -\infty$ | $\lim_{x\to+\infty} \frac{x^2+5x}{x^2+x-6} = 1$ |
| $\lim_{x\to2^+} \frac{x^2+5x}{x^2+x-6} = +\infty$ | |

Graph $f(x) = \frac{x^2+5x}{x^2+x-6}$



6. Compute: $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} =$

Step #1 Try Plugging in:

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \frac{\sqrt{(6)-2}-2}{(6)-6} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} = \lim_{x\to 6} \frac{\left(\sqrt{x-2}\right)^2-(2)^2}{(x-6)\left[\sqrt{x-2}+2\right]}$$

$$= \lim_{x\to 6} \frac{(x-2)-4}{(x-6)\left[\sqrt{x-2}+2\right]} = \lim_{x\to 6} \frac{(x-6)}{(x-6)\left[\sqrt{x-2}+2\right]} = \lim_{x\to 6} \frac{1}{\left[\sqrt{x-2}+2\right]}$$

$$= \frac{1}{\left[\sqrt{(6)-2}+2\right]} = \frac{1}{[2+2]} = \frac{1}{4}$$

i.e., $\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} = \frac{1}{4}$

7.

| $x =$ | $f(x) =$ |
|---|---|
| | |
| $-2.5$ | $-3.6$ |
| $-2.1$ | $-30.8$ |
| $-2.01$ | $-318.9$ |
| $-2.001$ | $-3,241.9$ |
| $-2.0001$ | $-35,342.2$ |

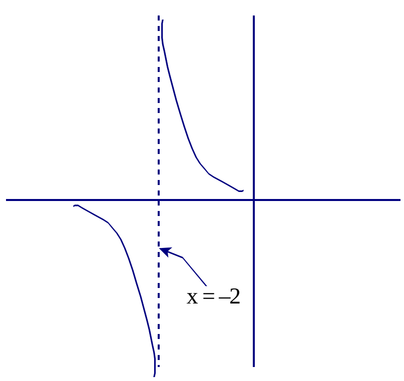
| $x =$ | $f(x) =$ |
|---|---|
| | |
| $-1.5$ | $3.6$ |
| $-1.9$ | $30.8$ |
| $-1.99$ | $318.9$ |
| $-1.999$ | $3,241.9$ |
| $-1.9999$ | $35,342.2$ |

Based on the information in the table above, do the following:

(a) $\lim_{x\to -2^-} f(x) = -\infty$

(b) $\lim_{x\to -2^+} f(x) = +\infty$

(c) Graph $f(x)$



8. Determine whether or not $f(x)$ is continuous at the point $x = 3$. (Justify your answer)

$$f(x) = \begin{cases} 3x+1 & \text{for } x < 3 \\ 10 & \text{for } x = 3 \\ x^2+1 & \text{for } x > 3 \end{cases}$$

First of all, let's recognize that $f(x)$ will be continuous at the point $x = 3$ exactly when $\lim_{x\to 3} f(x) = f(3)$.

So we should compute: $\lim_{x\to 3} f(x)$

The problem is that $f(x)$ is defined differently for $x < 3$ than it is for $x > 3$.

So we must compute the one sided limits as $x \to 3$

**Observe:** As $x \to 3^-$, $x < 3$.

Therefore: $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (3x+1) = 3(3)+1 = 10$

**Also:** As $x \to 3^+$, $x > 3$.

Therefore: $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x^2+1) = (3)^2 + 1 = 10$

Since the one-sided limits are equal, $\lim_{x\to 3} f(x)$ exists, and is equal to the common value of the one-sided limits.

i.e., $\lim_{x\to 3} f(x) = 10$

$\Rightarrow \lim_{x\to 3} f(x) = f(3)$

Hence, $f(x)$ is continuous at the point $x = 3$