# MTH 1125 (12 pm Class) - Test 2 - Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\frac{d}{dx}\left[2x^6+3x^4+2x^3+3x^2+3x+\sqrt{x}+6\right]=$

$\frac{d}{dx}\left[2x^6+3x^4+2x^3+3x^2+3x+x^{\frac{1}{2}}+6\right]=2\left[6x^5\right]+3\left[4x^3\right]+2\left[3x^2\right]+3\left[2x\right]+3+\left[\frac{1}{2}x^{-\frac{1}{2}}\right]$

$=12x^5+12x^3+6x^2+6x+3+\frac{1}{2}x^{-\frac{1}{2}}$

$$\text{i.e., } \frac{d}{dx}\left[2x^6+3x^4+2x^3+3x^2+3x+\sqrt{x}+6\right]=12x^5+12x^3+6x^2+6x+3+\frac{1}{2}x^{-\frac{1}{2}}$$

2. Compute: $\frac{d}{dx}\left[\left(5x^3+4x^2\right)\sec(x)\right]=$

$$\frac{d}{dx}\left[\underbrace{\left(5x^3+4x^2\right)}_{1^{st}}\underbrace{\sec(x)}_{2^{nd}}\right]=\underbrace{\left(15x^2+8x\right)}_{1^{\text{st}}\text{ prime}}\cdot\underbrace{\sec(x)}_{2^{nd}}+\underbrace{\sec(x)\tan(x)}_{2^{\text{nd}}\text{ prime}}\cdot\underbrace{\left(5x^3+4x^2\right)}_{1^{st}}$$

$$\frac{d}{dx}\left[\left(5x^3+4x^2\right)\sec(x)\right]=\left(15x^2+8x\right)\cdot\sec(x)+\sec(x)\tan(x)\cdot\left(5x^3+4x^2\right)$$

3. Compute: $\frac{d}{dx}\left[\frac{\tan(x)}{x^3+2x}\right]=$

$$\frac{d}{dx}\left[\frac{\overbrace{\tan(x)}^{\text{top}}}{\underbrace{x^3+2x}_{\text{Bottom}}}\right]=\frac{\overbrace{\sec^2(x)}^{\text{top prime}}\cdot\overbrace{\left(x^3+2x\right)}^{\text{bottom}}-\overbrace{\left(3x^2+2\right)}^{\text{bottom prime}}\cdot\overbrace{\tan(x)}^{\text{top}}}{\underbrace{\left(x^3+2x\right)^2}_{\text{bottom squared}}}$$

$$\text{i.e., } \frac{d}{dx}\left[\frac{\tan(x)}{x^3+2x}\right]=\frac{\sec^2(x)\left(x^3+2x\right)-\left(3x^2+2\right)\tan(x)}{\left(x^3+2x\right)^2}$$

4. Compute: $\frac{d}{dx}\left[\left(6x^5+10x^3+10\right)^{20}\right] =$ This is a function raised to a power.

$$\frac{d}{dx}\left[\left(6x^5+10x^3+10\right)^{20}\right] = \underbrace{20\left(6x^5+10x^3+10\right)^{19}}_{\substack{\text{power rule}\\ \text{as usual}}} \cdot \underbrace{\left(30x^4+30x^2\right)}_{\substack{\text{derivative}\\ \text{of inner}}}$$

i.e., $\frac{d}{dx}\left[\left(6x^5+10x^3+10\right)^{20}\right] = 20\left(6x^5+10x^3+10\right)^{19}\left(30x^4+30x^2\right)$

5. Given that $f(x) = 3x^2 - 2x + 3$, give the *equation* of the line tangent to the graph of $f(x)$ at the point $(2, 11)$.

We need two things:

i. A point on the line (We have that: $(x_1, y_1) = (2, 11)$)

ii. The slope of the line (This is $f'(x_1)$)

$f'(x) = 6x - 2$

The slope at the point $(x_1, y_1) = (2, 11)$ is $f'(2) = 6(2) - 2 = 10$

We will use the Point-Slope equation of a line: $y - y_1 = m(x - x_1)$

Thus, the equation of the line tangent to the graph of $f(x)$ is:

$y - 11 = 10(x - 2)$

The equation of the line tangent is $y - 11 = 10(x - 2)$

6. Given that $x = \sin(t)$ and that $t = 5y^3 - 2y$; compute $\frac{dx}{dy}$ **using the Liebniz form of the Chain Rule.** (In particular, when doing this exercise, write the Liebniz form of the chain rule, that you are going to use, explicitly.)

   We know:

   $\frac{dx}{dt} = \cos(t)$

   $\frac{dt}{dy} = 15y^2 - 2$

   We want: $\frac{dx}{dy}$

   By the Liebniz form of the Chain Rule:

   $$\frac{dx}{dy} = \frac{dx}{dt}\frac{dt}{dy} = \cos(t)\left(15y^2 - 2\right) = \underbrace{\cos\left(5y^3 - 2y\right)\left(15y^2 - 2\right)}_{\substack{\text{express solely in terms of} \\ \text{independent variable } y}}$$

   i.e. $\frac{dx}{dy} = \cos\left(5y^3 - 2y\right)\left(15y^2 - 2\right)$

7. Compute: $\frac{d}{dx}\left[\cos\left(8x^4\right)\right] =$

| | | |
|---|---|---|
| Outer: | = | $\cos(\quad)$ |
| Deriv. of outer | = | $-\sin(\quad)$ |

   $$\frac{d}{dx}\left[\begin{array}{c} \cos\left(8x^4\right) \\ \nearrow \quad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{-\sin\left(8x^4\right)}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{\left(32x^3\right)}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

   i.e., $\frac{d}{dx}\left[\cos\left(8x^4\right)\right] = -\sin\left(8x^4\right) \cdot \left(32x^3\right)$

8. Compute: $\frac{d}{dx}\left[\left(3x^2+6\right)^{10}\left(2x^3+6x\right)^5\right] =$

In the broadest sense, this is a PRODUCT - USE the PRODUCT RULE.

$$\frac{d}{dx}\left[\left(3x^2+6\right)^{10}\left(2x^3+6x\right)^5\right] = \underbrace{\underbrace{\left(\frac{d}{dx}\left[\left(3x^2+6\right)^{10}\right]\right)}_{1^{\text{st}}\text{ prime}}\underbrace{\left(2x^3+6x\right)^5}_{2^{\text{nd}}} + \underbrace{\left(\frac{d}{dx}\left[\left(2x^3+6x\right)^5\right]\right)}_{2^{\text{nd}}\text{ prime}}\underbrace{\left(3x^2+6\right)^{10}}_{1^{\text{st}}}}_{\text{Product Rule}}$$

$$= \overbrace{10\left(3x^2+6\right)^9}^{\text{Power Rule as Usual}}\ \overbrace{(6x)}^{\text{Deriv. of Inner}}\ \left[\left(2x^3+6x\right)^5\right] + \overbrace{5\left(2x^3+6x\right)^4}^{\text{Power Rule as Usual}}\overbrace{\left(6x^2+6\right)}^{\text{Deriv. of Inner}}\left[\left(3x^2+6\right)^{10}\right]$$

i.e., $\frac{d}{dx}\left[\left(3x^2+6\right)^{10}\left(2x^3+6x\right)^5\right] = 10\left(3x^2+6\right)^9(6x)\left(2x^3+6x\right)^5+5\left(2x^3+6x\right)^4\left(6x^2+6\right)\left(3x^2+6\right)^{10}$

9. Compute: $\frac{d}{dx}\left[\sin^3\left(5x^4+4x^3\right)\right] =$ Re-write!

$\frac{d}{dx}\left[\left(\sin\left(5x^4+4x^3\right)\right)^3\right] =$ This is a function, raised to a power

$$\frac{d}{dx}\left[\left(\sin\left(5x^4+4x^3\right)\right)^3\right] = \underbrace{3\left(\sin\left(5x^4+4x^3\right)\right)^2}_{\text{power rule as usual}} \cdot \underbrace{\left(\frac{d}{dx}\left[\sin\left(5x^4+4x^3\right)\right]\right)}_{\text{derivative of inner}}$$

$$= 3\left(\sin\left(5x^4+4x^3\right)\right)^2 \cdot \underbrace{\cos\left(5x^4+4x^3\right)\cdot\left(20x^3+12x^2\right)}_{\text{Chain Rule}}$$

i.e., $\frac{d}{dx}\left[\sin^3\left(5x^4+4x^3\right)\right] = 3\left(\sin\left(5x^4+4x^3\right)\right)^2\cdot\cos\left(5x^4+4x^3\right)\cdot\left(20x^3+12x^2\right)$

$= \left(60x^3+36x^2\right)\cdot\left(\sin\left(5x^4+4x^3\right)\right)^2\cdot\cos\left(5x^4+4x^3\right)$

10. Given that $f(x) = 2x^2 - 5x + 6$,compute $f'(x)$ **using the definition of derivative.**

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[2(x+\Delta x)^2 - 5(x+\Delta x)+6\right] - \left[2x^2 - 5x + 6\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[2\left(x^2+2x\Delta x+\Delta x^2\right) - 5(x+\Delta x)+6\right] - \left[2x^2 - 5x + 6\right]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\left[2x^2+4x\Delta x+2\Delta x^2 - 5x - 5\Delta x+6\right] - \left[2x^2 - 5x + 6\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 - 5\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\Delta x(4x+2\Delta x-5)}{\Delta x} = \lim_{\Delta x \to 0} 4x + 2\Delta x - 5 = 4x + 2(0) - 5 = 4x - 5$

i.e., $f'(x) = 4x - 5$

11. $3x^2 + 3y^2 = x^3y^4$. Compute $\frac{dy}{dx}$.

i. Differentiate both sides w.r.t. $x$

$\frac{d}{dx}\left[3x^2 + 3y^2\right] = \frac{d}{dx}\left[\underbrace{x^3}_{1^{\text{st}}}\underbrace{y^4}_{2^{\text{nd}}}\right]$

$\Rightarrow 6x + 6y \cdot \frac{dy}{dx} = \underbrace{3x^2}_{1^{\text{st}}\text{ prime}} \cdot \underbrace{y^4}_{2^{\text{nd}}} + \underbrace{4y^3 \cdot \frac{dy}{dx}}_{2^{\text{nd}}\text{ prime}} \cdot \underbrace{x^3}_{1^{\text{st}}}$

ii. Solve algebraically for $\frac{dy}{dx}$

a. Get $\frac{dy}{dx}$ terms on left side, all other terms on right side

$\Rightarrow 6y\frac{dy}{dx} - 4y^3\frac{dy}{dx}x^3 = 3x^2y^4 - 6x$

b. Factor out $\frac{dy}{dx}$

$\Rightarrow (6y - 4y^3x^3)\frac{dy}{dx} = 3x^2y^4 - 6x$

c. Divide both sides by the cofactor of $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{3x^2y^4 - 6x}{6y - 4y^3x^3}$

$\frac{dy}{dx} = \frac{3x^2y^4 - 6x}{6y - 4y^3x^3}$

Extra (Wow! 10 pts!) Given that $T'(x) = \frac{1}{1+x^2}$ (i.e., $\frac{d}{dx}[T(x)] = \frac{1}{1+x^2}$); compute $\frac{d}{dx}[T(\sec(x))]$

| | | |
|---|---|---|
| Outer: | $=$ | $T(\quad)$ |
| Deriv. of outer | $=$ | $\frac{1}{1+(\quad)^2}$ |

$$\frac{d}{dx}\left[\begin{array}{c} \frac{d}{dx}[T(\sec(x))] \\ \nearrow \qquad \uparrow \\ \text{outer} \quad \text{inner} \end{array}\right] = \underbrace{\frac{1}{1+(\sec(x))^2}}_{\substack{\text{Deriv of outer,} \\ \text{eval. at inner}}} \cdot \underbrace{(\sec(x)\tan(x))}_{\substack{\text{deriv. of} \\ \text{inner}}}$$

i.e., $\frac{d}{dx}[T(\sec(x))] = \frac{\sec(x)\tan(x)}{1+\sec^2(x)}$