History of Math - Reading Assignment #1 - Answers

Term V - 2024

Pat Rossi Name ____________________

Instructions. Read pages 1-28, find the answers to these questions in your reading.

1. In what region(s) did special symbols for numbers first appear?

   In the river valleys of the Nile, Tigris-Euphrates, Indus, and Yangtze.

2. What is the relationship between number systems and language?

   There is *unmistakable evidence* that the written languages of these cultures grew out of their previously written number systems.

3. What circumstances precipitated the development of Egyptian mathematics?

   The unification of Egypt under a single leader and the emergence of an extensive administrative system. The taking of a census, the levying and collecting of taxes, the formation and maintenance of an army, etc. required reckoning with large numbers.

   Remark: This is a classic example of "the theory coming from the application."

4. By what date did the Egyptians have a "fully developed number system," that would allow for counting to continue indefinitely?

   By 3500 B.C., had a number system that would allow counting to continue indefinitely.

5. What examples from archaeology confirm that the Egyptians did indeed have a "fully developed number system?"

   The **macehead of King Narmer** and the **Narmer Palette** (a slate palette) both document the Egyptians victory over the Libyans - "taking 120,000 prisoners, 400,000 oxen and 1,422,000 goats."

   The **Book of the Dead** mentions the population of Egypt to be "four million, six hundred and one thousand, and two hundred."

6. Describe the Egyptian system (i.e., what base, was it place value?, etc.)

   The Egyptian system was **decimal (i.e., base 10),** with unique numerals for each power of 10 from 1 (ten to the zero) to 10,000,000. The Egyptian was also **additive** (i.e., the number represented by the numerals was the sum of the values represented by the individual numerals).

7. Describe the Ionic system (i.e., what base, was it place value?, etc.)

   The Ionic system was **decimal (i.e., base 10),** and **additive** (i.e., the number represented by the symbols was the sum of the numbers represented by the individual symbols). The system had unique symbols for every multiple of 1, 10, and 100. Larger numbers were written using an accent mark (below, left) as a multiplier of 1000 and an "M" (below or to the right) as a multiplier of 10,000.

8. Why did the multiplicity of symbols in the Ionic system hinder the Greeks of Ionia in their mathematical development?

   The fact that there were 27 different symbols used to represent the different multiples of powers of 10 (9 multiples of $10^0 = 1$; 9 multiples of $10^1 = 10$; 9 multiples of $10^2 = 100$) meant that the Greeks would have had to know 729 different multiplication facts in order to perform multiplication. The sheer magnitude of the number of multiplication facts tended to obscure relationships that might have been obvious, otherwise. (For example, the fact that a natural number is even exactly when it has a "ones digit" of 0, 2, 4, 6, 8; or that a natural number is a multiple of 5 exactly when it has a "ones digit" of 0 or 5.)

9. Regarding the Babylonian system, why was Cuneiform script a natural consequence of the writing medium?

   Because clay was the writing medium and required a sharp-edged "stylus" could easily make only linear, "vertical" strokes, while the base of the stylus could easily make only a "wedge-shaped" impression. Cuneiform script comprised symbols that were a combination of these two shapes.

10. Describe the Babylonian system (i.e., what base, was it place value?, etc.)

    The Babylonian system was **sexagesimal (i.e., base 60),** and was, **in some ways, additive** and **in some ways, positional.** It was **additive** in the sense that the coefficients of powers of 60 were written additively and it was **positional** in the sense that the coefficients of the larger powers of 60 appeared to the left of the coefficients of the smaller powers of 60.

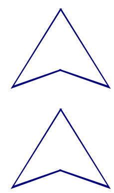
11. What flaw (causing ambiguity in interpreting the value of a number) existed in the Babylonian system?

    The Babylonian system lacked a symbol for zero. Thus, since the system was partly positional (coefficients larger powers of 60 appeared to the left of the coefficients of the smaller powers of 60), there was no way of denoting that the representation of a number did not contain coefficients of a specific power of 60.

12. How and when was this flaw corrected?

    A physical "gap" between coefficients of powers of 60 was used to indicate that the representation of the number did not contain the coefficient of a specific power of 60. This was not wholly satisfactory.

    From 300 B.C. onward, a separate symbol called a divider (shown below) was used as a "place holder" to indicate that the representation of the number did not contain coefficient of a specific power of 60.



13. Did this remedy the situation completely?

    No, the divider symbol was only used *between* coefficients of powers of 60. They had no way of indicating that there was no coefficient of 60^0 (i.e. They had no way of indicating that there was no coefficient of 1).

14. Relevant to missing coefficients of powers of 60, what innovation did Ptolemy make circa 150 AD?

    Ptolemy began using o (omicron) "as a zero" in positions where the corresponding power of 60 had no nonzero coefficient. However, Ptolemy did not use the symbol omicron computationally.

15. What do we know about the early Chinese number system?

    In the middle of the second millennium B.C., the Chinese were already keeping track of astronomical events on bone fragments.

    By 1400 B.C., the Chinese had a positional numeration system that used nine symbols.

16. Why is so little known about the number systems of India and China, as compared with Egypt and Babylonia?

    The dry climate and soil of Egypt an Babylonia preserved writing media that would long since have perished in more moist climates.

    The writing media of the Chinese consisted of material made of silk or of bamboo strips - both of which deteriorated over time due to climate and nature.

    Also, during the course of political upheavals in China, despotic rulers attempted (and nearly succeeded) to destroy all books.

17. Throughout Chinese history, what has been the main importance of mathematics?

    The construction of the calendar. Their agricultural economy was depended on artificial irrigation, so it was necessary to be forewarned of the beginning and end of the rainy monsoon season, as well as of the melting snows and the consequent rise of the rivers.

On page 18, do the following exercises: 1 a, c, e; 2 a, b; 3 a, c; 4; 5 a, c, e; 6 a, c; 7 a, c

On page 28, do the following exercises: 1 d, e, f; 2 a, b (ans 116,712); 3-9