MTH 1126 Practice Test #5 - Answers

Fall 2015

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Instructions. In exercises 1 - 9 determine whether the given series converges or diverges.

1. i.e., $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ converges by the Alternating Series Test.

2. i.e., $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n)!}$ converges by the Alternating Series Test.

3. i.e., $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n+4}$ diverges because $a_n \nrightarrow 0$

4. i.e., $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\ln(n+2)}$ converges by the Alternating Series Test.

5. i.e., $\sum_{n=1}^{\infty}\frac{n!}{(2n)!}$ converges by the Ratio Test.

6. i.e., $\sum_{n=1}^{\infty}(-1)^{n}\frac{(2n-1)!}{e^n}$ diverges by the Ratio Test.

7. i.e., $\sum_{n=1}^{\infty}n^2\left(\frac{3}{7}\right)^n$ converges by the n^{th} Root Test.

8. i.e., $\sum_{n=1}^{\infty}\left(\frac{1}{2}+\frac{1}{n}\right)^n$ converges by the n^{th} Root Test.

9. i.e., $\sum_{n=1}^{\infty}n^n\left(\frac{3}{5}\right)^n$ diverges by the n^{th} Root Test.

In exercises 10 - 12, determine whether the given series is divergent, conditionally convergent, or absolutely convergent.

10.

i.e., $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$ converges absolutely.

11.

i.e., $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n+1)}$ converges conditionally.

12.

i.e., $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n)!}$ converges absolutely.

In problems 13 - 15 simplify (identify) the given expression.

13.

i.e., $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots = e^x$

14.

i.e., $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sin(x)$

15.

i.e., $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \cos(x)$

16.

i.e., $\sin(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3 + \frac{\sqrt{2}}{48}\left(x - \frac{\pi}{4}\right)^4 + \ldots$

17.

i.e., $\frac{1}{x} = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \frac{1}{32}(x-c)^4 + \ldots$

18.

i.e., $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + \frac{(-1)^{n+1}x^n}{n} + \ldots$

In problems 19 - 20 use a known Taylor Series expansion to derive an expansion for the given function.

19. i.e., $\frac{1-\cos(x)}{x} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{(2n)!} = \frac{x}{2!} - \frac{x^3}{4!} + \frac{x^5}{6!} - \ldots$

20. i.e., $\cos\left(x^2\right) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!} = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \ldots$