MTH 6610 History of Math - Midterm Exam

Term 5, 2024

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Instructions. Show CLEARLY how you arrive at your answers

1. Using dot diagrams, show that the sum of consecutive triangular numbers is a square number (in particular, show that $t_{n-1} + t_n = s_n$).

2. Using dot diagrams, show that the sum of the first n odd natural numbers is equal to n^2.

   i.e., show that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

3. Using dot diagrams, derive a formula for the value of the n^{th} triangular number, t_n.

4. The n^{th} oblong number, denoted o_n, is defined to be number of dots in a rectangular array having $n+1$ rows and n columns. (See Exercise 7 on p. 103 of our text.) Note that $o_n = n\,(n+1)$. Prove algebraically and geometrically (i.e., using "dot diagrams") that the n^{th} oblong number o_n is the sum of two equal triangular numbers.

5. Prove algebraically and geometrically (i.e., using “dot diagrams”) that $4o_n + 1 = s_{2n+1}$.

6. Illustrate the Babylonian method for generating Pythagorean Triples with three examples

7. Illustrate the Pythagoreans' method for generating Pythagorean Triples with three examples

8. Illustrate Plato's method for generating Pythagorean Triples with three examples.

From exercises 9-10, select one.

9. Prove that $\sqrt{2}$ is irrational.

10. Prove that the side and diagonal of a square are “incommensurable” (not of common measure)

11. Show (and explain) how Thales measured the height of the Great Pyramid.

From the Homework

12. Convert from Hindu-Arabic (our number system) to Egyptian hieroglyphics:

    (a) 2614

    (b) 388

13. Convert the Egyptian numbers to Arabic (our number system).

    (a) ‘

14. Write the (Ionian) Greek numerals corresponding to:

    (a) 2643

    (b) 876

15. Solve this equation by the "Egyptian method." (i.e. False Position) $x + \frac{1}{5}x = 14$

16. Solve this equation by the "Egyptian method." (i.e. Double False Position) $6x+8=0$

17. Using the 2/n table, write $\frac{4}{11}$ as the sum of unit fractions (with no repetition)

18. Represent $\frac{3}{7}$ as the sum of distinct unit fractions, by using the splitting identity

19. Represent $\frac{3}{7}$ as the sum of distinct unit fractions, by using Fibonacci's method.

For problems 20 - 21, use “Egyptian methods” to compute:

20. $47 \div 8$ (Do this out “long division” the whole way)

21. $26 \cdot 33$

22. Verify (prove) that $(3, 4, 5)$ is the only Pythagorean Triple involving consecutive positive numbers.

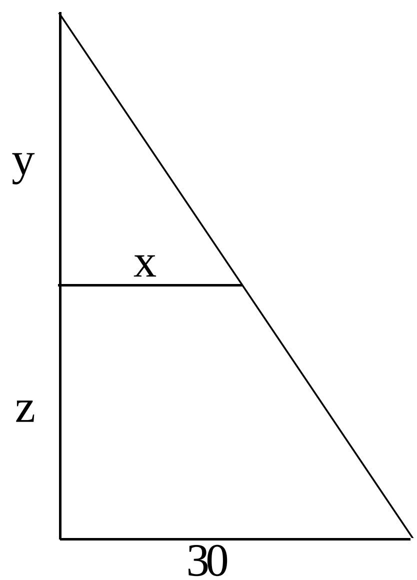
23. With the natural numbers from 1 to n as arranged below, derive a well known formula by adding the columns and adding the rows.

$$\begin{array}{cccccc} 1 & 2 & 3 & \dots & n-1 & n \\ n & n-1 & n-2 & \dots & 2 & 1 \end{array}$$

24. A triangle whose base has length 30 is divided into two parts by a line segment drawn parallel to its base. It is given that the resulting right trapezoid has an area larger by 7,0 = 420 than the upper triangle, and that the difference between the height y of the upper triangle and the height z of the trapezoid is 20. If x is the length of the upper base of the trapezoid these statements lead to the equations:

$$\frac{1}{2}z\,(x+30) = \frac{1}{2}xy + 420; \qquad y - z = 20$$

Find the quantities x, y, z using equivalent triangles.



25. Given that the circumference of a circle is 60 units and the length of a perpendicular from the center of a chord of the circle to the circumference is 2 units, find the length of the chord. In solving the problem, use $\pi = 3$, as did the Babylonians.