MTH 1125 - Test #1

FALL 2007

Pat Rossi Name ______________________

Instructions. You may NOT use calculators.

Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} =$

   1. Try Plugging in:

      $\lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} = \frac{(3)^2-(3)-6}{(3)^2-5(3)+6} = \frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

      $\lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} = \lim_{x\to 3} \frac{(x+2)(x-3)}{(x-2)(x-3)} = \lim_{x\to 3} \frac{(x+2)}{(x-2)} = \frac{(3)+2}{(3)-2} = \frac{5}{1} = 5$

      $\boxed{\text{i.e., } \lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} = 5}$

2. Compute: $\lim_{x\to 1} \frac{x^2+1}{x^2-x-5} =$

   1. Try Plugging in:

      $\lim_{x\to 1} \frac{x^2+1}{x^2-x-5} = \frac{(1)^2+1}{(1)^2-(1)-5} = \frac{2}{-5} = -\frac{2}{5}$

      $\boxed{\text{i.e., } \lim_{x\to 1} \frac{x^2+1}{x^2-x-5} = -\frac{2}{5}}$

3. Compute: $\lim_{x\to 5} \frac{\sqrt{11+x}-4}{x-5} =$

   1. Try Plugging in:

      $\lim_{x\to 5} \frac{\sqrt{11+x}-4}{x-5} = \frac{\sqrt{11+(5)}-4}{(5)-5} = \frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

      $\lim_{x\to 5} \frac{\sqrt{11+x}-4}{x-5} = \lim_{x\to 5} \frac{\sqrt{11+x}-4}{x-5} \cdot \frac{\sqrt{11+x}+4}{\sqrt{11+x}+4} = \lim_{x\to 5} \frac{\left(\sqrt{11+x}\right)^2-(4)^2}{(x-5)\left[\sqrt{11+x}+4\right]} =$

      $\lim_{x\to 5} \frac{(11+x)-16}{(x-5)\left[\sqrt{11+x}+4\right]} = \lim_{x\to 5} \frac{x-5}{(x-5)\left[\sqrt{11+x}+4\right]} = \lim_{x\to 5} \frac{1}{\left[\sqrt{11+x}+4\right]} =$

      $= \frac{1}{\left[\sqrt{11+(5)}+4\right]} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}$

      $\boxed{\text{i.e., } \lim_{x\to 5} \frac{\sqrt{11+x}-4}{x-5} = \frac{1}{8}}$

4. Compute: $\lim_{x\to 4} \frac{x^2+2x+1}{x^2-6x+8} =$

1. Try Plugging in:

   $\lim_{x\to 4} \frac{x^2+2x+1}{x^2-6x+8} = \frac{(4)^2+2(4)+1}{(4)^2-6(4)+8} = \frac{25}{0}$ No Good - Zero Divide!

2. Try Factoring and Cancelling:

   No Good - Cancelling will only work when Step #1 yields $\frac{0}{0}$.

3. Evaluate the one-sided limits:

   $\lim_{x\to 4^-} \frac{x^2+2x+1}{x^2-6x+8} = \lim_{x\to 4^-} \frac{x^2+2x+1}{(x-2)(x-4)} = \frac{25}{(2)(-\varepsilon)} = \frac{\left(\frac{25}{2}\right)}{(-\varepsilon)} = -\infty$

   $$\begin{array}{|ll|} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \\ \end{array}$$

   $\lim_{x\to 4^+} \frac{x^2+2x+1}{x^2-6x+8} = \lim_{x\to 4^+} \frac{x^2+2x+1}{(x-2)(x-4)} = \frac{25}{(2)(\varepsilon)} = \frac{\left(\frac{25}{2}\right)}{(\varepsilon)} = +\infty$

   $$\begin{array}{|ll|} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \\ \end{array}$$

   Since the one-sided limits are not equal, $\lim_{x\to 4} \frac{x^2+2x+1}{x^2-6x+8}$ Does Not Exist.

5. $f(x) = \frac{x^2+1}{x^2-1}$ Find the asymptotes and graph.

Verticals Look for those x-values that cause division by zero.

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow (x+1)(x-1) = 0$

$\Rightarrow x = -1$ and $x = 1$ are *possible* vertical asymptotes.

Compute the one-sided limits of $f(x)$, as x approaches -1 and as as x approaches 1.

$\lim_{x\to -1^-} \frac{x^2+1}{x^2-1} = \lim_{x\to -1^-} \frac{x^2+1}{(x+1)(x-1)} = \frac{2}{(-\varepsilon)(-2)} = \frac{1}{\varepsilon} = +\infty$

$$\begin{array}{|ll|} & x \to -1^- \\ \Rightarrow & x < -1 \\ \Rightarrow & x + 1 < 0 \\ \end{array}$$

↖ Infinite limits indicate

that $x = -1$ IS a vertical asymptote ↙

$\lim_{x\to -1^+} \frac{x^2+1}{x^2-1} = \lim_{x\to -1^+} \frac{x^2+1}{(x+1)(x-1)} = \frac{2}{(\varepsilon)(-2)} = \frac{-1}{\varepsilon} = -\infty$

$$\begin{array}{|ll|} & x \to -1^+ \\ \Rightarrow & x > -1 \\ \Rightarrow & x + 1 > 0 \\ \end{array}$$

$\lim_{x\to 1^-} \frac{x^2+1}{x^2-1} = \lim_{x\to 1^-} \frac{x^2+1}{(x+1)(x-1)} = \frac{2}{(2)(-\varepsilon)} = \frac{1}{-\varepsilon} = -\infty$

	$x \to 1^-$
$\Rightarrow$	$x < 1$
$\Rightarrow$	$x - 1 < 0$

$\nwarrow$ Infinite limits indicate

that $x = 1$ IS a
$\swarrow$ vertical asymptote

$\lim_{x\to 1^+} \frac{x^2+1}{x^2-1} = \lim_{x\to 1^+} \frac{x^2+1}{(x+1)(x-1)} = \frac{2}{(2)(\varepsilon)} = \frac{1}{\varepsilon} = +\infty$

	$x \to 1^+$
$\Rightarrow$	$x > 1$
$\Rightarrow$	$x - 1 > 0$

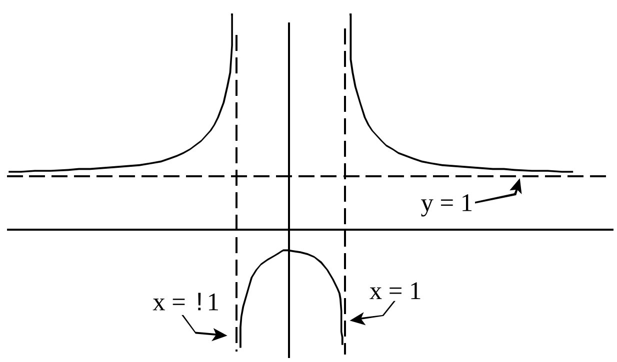
Horizontals Compute the limits as $x \to \pm\infty$

$\lim_{x\to -\infty} \frac{x^2+1}{x^2-1} = \lim_{x\to -\infty} \frac{x^2}{x^2} = \lim_{x\to -\infty} (1) = 1$

$\nwarrow$ Finite, constant limits indicate
that $y = 1$ IS a
$\swarrow$ horizontal asymptote

$\lim_{x\to +\infty} \frac{x^2+1}{x^2-1} = \lim_{x\to +\infty} \frac{x^2}{x^2} = \lim_{x\to +\infty} (1) = 1$

Graph $f(x) = \frac{x^2+1}{x^2-1}$



6. ~

x	$f(x)$
1.0	6.17
1.5	88.36
1.9	978.78
1.99	9968.12
1.999	124877.79

x	$f(x)$
3.0	-6.17
2.5	-88.36
2.1	-978.78
2.01	-9968.12
2.001	-124877.79

(a) $\lim_{x\to 2^-} f(x) = +\infty$ (as x approaches 2 through values less than 2, $f(x)$ gets unboundedly large in the positive direction.)

(b) $\lim_{x\to 2^+} f(x) = -\infty$ (as x approaches 2 through values greater than 2, $f(x)$ gets unboundedly large in the negative direction.)

(c) Sketch a graph of $f(x)$