# MTH 1125 - Test #1 Solutions

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**Instructions.** Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^2+x-6}{x^2-5x+6} =$

   (a) 1. Try Plugging in:

   $\lim_{x\to 2}\frac{x^2+x-6}{x^2-5x+6} = \frac{(2)^2+(2)-6}{(2)^2-5(2)+6} = \frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

   $\lim_{x\to 2}\frac{x^2+x-6}{x^2-5x+6} = \lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x-3)} = \lim_{x\to 3}\frac{(x+3)}{(x-3)} = \frac{(2)+3}{(2)-3} = -5$

   $\boxed{\text{i.e., } \lim_{x\to 2}\frac{x^2+x-6}{x^2-5x+6} = -5}$

2. Compute: $\lim_{x\to 2}\frac{x^2-x-6}{3x^2+1} =$

   (a) 1. Try Plugging in:

   $\lim_{x\to 2}\frac{x^2-x-6}{3x^2+1} = \frac{(2)^2-(2)-6}{3(2)^2+1} = -\frac{4}{13}$

   $\boxed{\text{i.e., } \lim_{x\to 2}\frac{x^2-x-6}{3x^2+1} = -\frac{4}{13}}$

3. Compute: $\lim_{x\to 5}\frac{\sqrt{11+x}-4}{x-5} =$

   (a) 1. Try Plugging in:

   $\lim_{x\to 5}\frac{\sqrt{11+x}-4}{x-5} = \frac{\sqrt{11+(5)}-4}{(5)-5} = \frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

   $\lim_{x\to 5}\frac{\sqrt{11+x}-4}{x-5} = \lim_{x\to 5}\frac{\sqrt{11+x}-4}{x-5}\cdot\frac{\sqrt{11+x}+4}{\sqrt{11+x}+4} = \lim_{x\to 5}\frac{\left(\sqrt{11+x}\right)^2-(4)^2}{(x-5)\left[\sqrt{11+x}+4\right]} =$

   $\lim_{x\to 5}\frac{(11+x)-16}{(x-5)\left[\sqrt{11+x}+4\right]} = \lim_{x\to 5}\frac{x-5}{(x-5)\left[\sqrt{11+x}+4\right]} = \lim_{x\to 5}\frac{1}{\left[\sqrt{11+x}+4\right]} =$

   $= \frac{1}{\left[\sqrt{11+(5)}+4\right]} = \frac{1}{8}$

   $\boxed{\text{i.e., } \lim_{x\to 5}\frac{\sqrt{11+x}-4}{x-5} = \frac{1}{8}}$

4. Compute: $\lim_{x\to 3} \frac{x^2+4}{x^2-x-6} =$

(a)
1. Try Plugging in:

   $\lim_{x\to 3} \frac{x^2+4}{x^2-x-6} = \frac{(3)^2+4}{(3)^2-(3)-6} = \frac{13}{0}$ No Good - Zero Divide!

2. Try Factoring and Cancelling:

   No Good - Cancelling will only work when Step #1 yields $\frac{0}{0}$.

3. Evaluate the one-sided limits:

   $\lim_{x\to 3^-} \frac{x^2+4}{x^2-x-6} = \lim_{x\to 3^-} \frac{x^2+4}{(x+2)(x-3)} = \frac{13}{(5)(-\varepsilon)} = \frac{\left(\frac{13}{5}\right)}{-\varepsilon} = -\infty$

   $\lim_{x\to 3^+} \frac{x^2+4}{x^2-x-6} = \lim_{x\to 3^+} \frac{x^2+4}{(x+2)(x-3)} = \frac{13}{(5)(\varepsilon)} = \frac{\left(\frac{13}{5}\right)}{\varepsilon} = +\infty$

   Since the one-sided limits are not equal, $\lim_{x\to 3} \frac{x^2+4}{x^2-x-6}$ Does Not Exist.

5. $f(x) = \frac{3x^2}{x^2+2x+1}$ Find the asymptotes and graph.

**Verticals** Look for those $x$-values that cause division by zero.

$\Rightarrow x^2 + 2x + 1 = 0$

$\Rightarrow (x+1)(x+1) = 0$

$\Rightarrow x = -1$ is a *possible* vertical asymptote.

Compute the one-sided limits of $f(x)$, as $x$ approaches $-1$.

$\lim_{x\to -1^-} \frac{3x^2}{x^2+2x+1} = \lim_{x\to -1^-} \frac{3x^2}{(x-1)^2} = \frac{3}{(-\varepsilon)^2} = \frac{3}{\varepsilon^2} = +\infty$

↖ Infinite limits indicate that $x = -1$ IS a vertical asymptote ↙

$\lim_{x\to -1^+} \frac{3x^2}{x^2+2x+1} = \lim_{x\to -1^+} \frac{3x^2}{(x-1)^2} = \frac{3}{(\varepsilon)^2} = \frac{3}{\varepsilon^2} = +\infty$

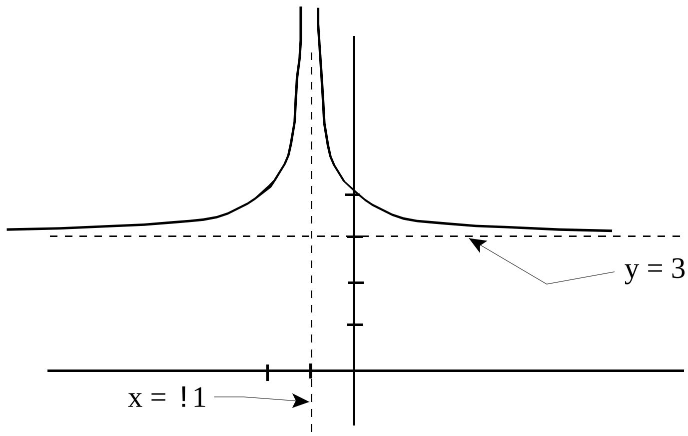
**Horizontals** Compute the limits as $x \to \pm\infty$

$\lim_{x\to -\infty} \frac{3x^2}{x^2+2x+1} = \lim_{x\to -\infty} \frac{3x^2}{x^2} = \lim_{x\to -\infty} 3 = 3$

↖ Finite, constant limits indicate that $y = 3$ IS a horizontal asymptote ↙

$\lim_{x\to +\infty} \frac{3x^2}{x^2+2x+1} = \lim_{x\to +\infty} \frac{3x^2}{x^2} = \lim_{x\to +\infty} 3 = 3$

Graph $f(x) = \frac{3x^2}{x^2+2x+1}$




$f(x) = \frac{3x^2}{x^2+2x+1}$

6. ~

| | $x$ | $f(x)$ | |
|---|---|---|---|
| $x$ | 1.0 | $-3.25$ | $f(x)$ |
| $\downarrow$ | 1.5 | $-56.789$ | $\downarrow$ |
| $2^-$ | 1.9 | $-488.78$ | $-\infty$ |
| | 1.99 | $-7768.12$ | |
| | 1.999 | $-15877.79$ | |

| | $x$ | $f(x)$ | |
|---|---|---|---|
| $x$ | 3.0 | 6.25 | $f(x)$ |
| $\downarrow$ | 2.5 | 96.789 | $\downarrow$ |
| $2^+$ | 2.1 | 788.78 | $+\infty$ |
| | 2.01 | 12768.12 | |
| | 2.001 | 45877.79 | |

(a) $\lim_{x\to 2^-} f(x) = -\infty$

(b) $\lim_{x\to 2^+} f(x) = +\infty$

(c) Sketch a graph of $f(x)$