MTH 1125 Test #1 - Solutions

Fall 2013 12 pm class

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Instructions. Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 2}\frac{x^3-2}{x^2+4}=$

Step #1 Try Plugging In:

$$\lim_{x\to 2}\frac{x^3-2}{x^2+4}=\frac{(2)^3-2}{(2)^2+4}=\frac{6}{8}=\frac{3}{4}$$

$$\boxed{\text{i.e., } \lim_{x\to 2}\frac{x^3-2}{x^2+4}=\frac{3}{4}}$$

2. Compute: $\lim_{x\to 5}\frac{x^2-4x-5}{x^2-3x-10}=$

Step #1 Try Plugging in:

$$\lim_{x\to 5}\frac{x^2-4x-5}{x^2-3x-10}=\frac{(5)^2-4(5)-5}{(5)^2-3(5)-10}=\frac{0}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

$$\lim_{x\to 5}\frac{x^2-4x-5}{x^2-3x-10}=\lim_{x\to 5}\frac{(x-5)(x+1)}{(x-5)(x+2)}=\lim_{x\to 5}\frac{(x+1)}{(x+2)}=\frac{(5)+1}{(5)+2}=\frac{6}{7}$$

$$\boxed{\text{i.e., } \lim_{x\to 5}\frac{x^2-4x-5}{x^2-3x-10}=\frac{6}{7}}$$

3. Compute: $\lim_{x\to 4}\frac{x-5}{x^2-5x+4}=$

Step #1 Try Plugging in:

$$\lim_{x\to 4}\frac{x-5}{x^2-5x+4}=\frac{(4)-5}{(4)^2-5(4)+4}=\frac{-1}{0}$$

No Good - Zero Divide!

Step #2 Try Factoring and Cancelling:

No Good!. "Factoring and Cancelling" only works when Step #1 yields $\frac{0}{0}$.

Step #3 Analyze the one-sided limits:

$$\lim_{x\to 4^-} \frac{x-5}{x^2-5x+4} = \lim_{x\to 4^-} \frac{x-5}{(x-1)(x-4)} = \frac{-1}{(3)(-\varepsilon)} = \frac{\left(-\frac{1}{3}\right)}{(-\varepsilon)} = +\infty$$

$$\begin{array}{ll} & x \to 4^- \\ \Rightarrow & x < 4 \\ \Rightarrow & x - 4 < 0 \end{array}$$

$$\lim_{x\to 4^+} \frac{x-5}{x^2-5x+4} = \lim_{x\to 4^+} \frac{x-5}{(x-1)(x-4)} = \frac{-1}{(3)(+\varepsilon)} = \frac{\left(-\frac{1}{3}\right)}{(+\varepsilon)} = -\infty$$

$$\begin{array}{ll} & x \to 4^+ \\ \Rightarrow & x > 4 \\ \Rightarrow & x - 4 > 0 \end{array}$$

Since the one-sided limits are not equal, $\lim_{x\to 4} \frac{x-5}{x^2-5x+4}$ **Does Not Exist!**

4. $f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{for } x < 3 \\ 2x - 1 & \text{for } x \geq 3 \end{cases}$ Determine whether or not $f(x)$ is continuous at the point $x = 3$. (Justify your answer.)

If $f(x)$ is continuous at the point $x = 3$, then $\lim_{x\to 3} f(x) = f(3)$.

To see if this is true, we'll compute $\lim_{x\to 3} f(x)$.

Since the definiton of $f(x)$ changes at $x = 3$, we must compute the one-sided limits in order to determine whether the limit exists.

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \lim_{x\to 3^-} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3^-} (x+3) = (3) + 3 = 6$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x-1) = 2(3) - 1 = 5$

Since the one-sided limits are NOT equal, $\lim_{x\to 3} f(x)$ Does Not Exist.

Hence: $\lim_{x\to 3} f(x) \neq f(3)$

Therefore, $f(x)$ is NOT continuous at $x = 3$

5. $f(x) = \frac{x^2+x-6}{x^2-x-6}$ Find the asymptotes and graph

Verticals

1. Find x-values that cause division by zero.

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3$ and $x = -2$ are possible vertical asymptotes.

2. Compute the one-sided limits.

$$\lim_{x\to -2^-} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to -2^-} \frac{x^2+x-6}{(x-3)(x+2)} = \frac{-4}{(-5)(-\varepsilon)} = \frac{-4}{(5)(\varepsilon)} = \frac{\left(-\frac{4}{5}\right)}{\varepsilon} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to -2^- \\ \Rightarrow & x < -2 \\ \Rightarrow & x+2 < 0 \end{array}}$$

$$\lim_{x\to -2^+} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to -2^+} \frac{x^2+x-6}{(x-3)(x+2)} = \frac{-4}{(-5)(\varepsilon)} = \frac{\left(\frac{4}{5}\right)}{\varepsilon} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to -2^+ \\ \Rightarrow & x > -2 \\ \Rightarrow & x+2 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = -2$ *is a vertical asymptote.*

$$\lim_{x\to 3^-} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to 3^-} \frac{x^2+x-6}{(x-3)(x+2)} = \frac{6}{(-\varepsilon)(5)} = \frac{\left(\frac{6}{5}\right)}{(-\varepsilon)} = -\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & x-3 < 0 \end{array}}$$

$$\lim_{x\to 3^+} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to 3^+} \frac{x^2+x-6}{(x-3)(x+2)} = \frac{6}{(\varepsilon)(5)} = \frac{\left(\frac{6}{5}\right)}{(\varepsilon)} = +\infty$$

$$\boxed{\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & x-3 > 0 \end{array}}$$

Since the one-sided limits are infinite, $x = 3$ *is a vertical asymptote.*

Horizontals

Compute the limits as $x \to -\infty$ and as $x \to +\infty$

$\lim_{x\to-\infty} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to-\infty} \frac{x^2}{x^2} = \lim_{x\to-\infty} 1 = 1$

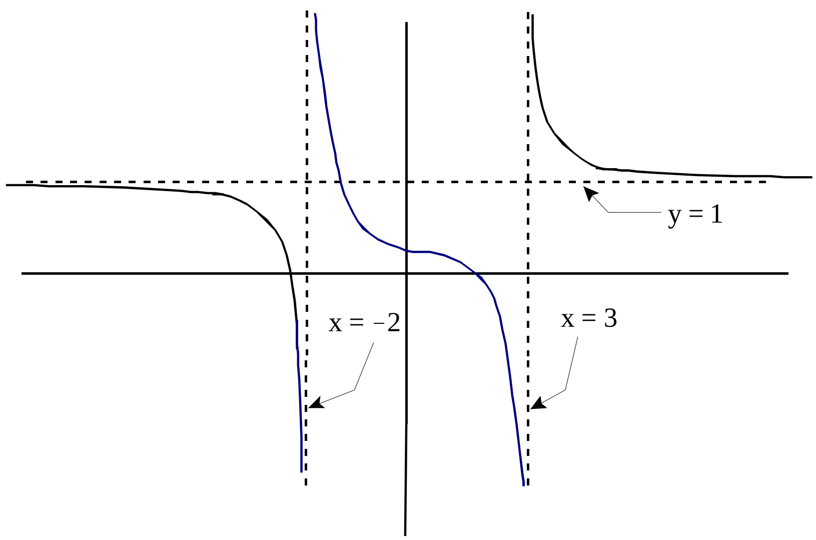
$\lim_{x\to+\infty} \frac{x^2+x-6}{x^2-x-6} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$

Since the limits are finite and constant, $y = 1$ is a horizontal asymptote.

Summary:

$\lim_{x\to-2^-} \frac{x^2+x-6}{x^2-x-6} = -\infty$	
$\lim_{x\to-2^+} \frac{x^2+x-6}{x^2-x-6} = +\infty$	$\lim_{x\to-\infty} \frac{x^2+x-6}{x^2-x-6} = 1$
$\lim_{x\to3^-} \frac{x^2+x-6}{x^2-x-6} = -\infty$	$\lim_{x\to+\infty} \frac{x^2+x-6}{x^2-x-6} = 1$
$\lim_{x\to3^+} \frac{x^2+x-6}{x^2-x-6} = +\infty$	

Graph $f(x) = \frac{x^2+x-6}{x^2-x-6}$



6. Compute: $\lim_{x\to 5} \frac{\sqrt{x+44}-7}{x-5} =$

Step #1 Try Plugging in:

$$\lim_{x\to 5} \frac{\sqrt{x+44}-7}{x-5} = \frac{\sqrt{(5)+44}-7}{(5)-5} = \frac{0}{0}$$

No Good -
Zero Divide!

Step #2 Try Factoring and Canceling:

$$\lim_{x\to 5} \frac{\sqrt{x+44}-7}{x-5} = \lim_{x\to 5} \frac{\sqrt{x+44}-7}{x-5} \cdot \frac{\sqrt{x+44}+7}{\sqrt{x+44}+7} = \lim_{x\to 5} \frac{\left(\sqrt{x+44}\right)^2-(7)^2}{(x-5)\left[\sqrt{x+44}+7\right]}$$

1. $= \lim_{x\to 5} \frac{(x+44)-49}{(x-5)\left[\sqrt{x+44}+7\right]} = \lim_{x\to 5} \frac{(x-5)}{(x-5)\left[\sqrt{x+44}+7\right]}$

   $= \lim_{x\to 5} \frac{1}{\left[\sqrt{x+44}+7\right]} = \frac{1}{\left[\sqrt{(5)+44}+7\right]} = \frac{1}{[7+7]} = \frac{1}{14}$

   i.e., $\lim_{x\to 5} \frac{\sqrt{x+44}-7}{x-5} = \frac{1}{14}$

7.

$x =$	$f(x) =$
2.5	10.1
2.9	100.8
2.99	$1,000.3$
2.999	$10,000.3$
2.9999	$100,000.9$

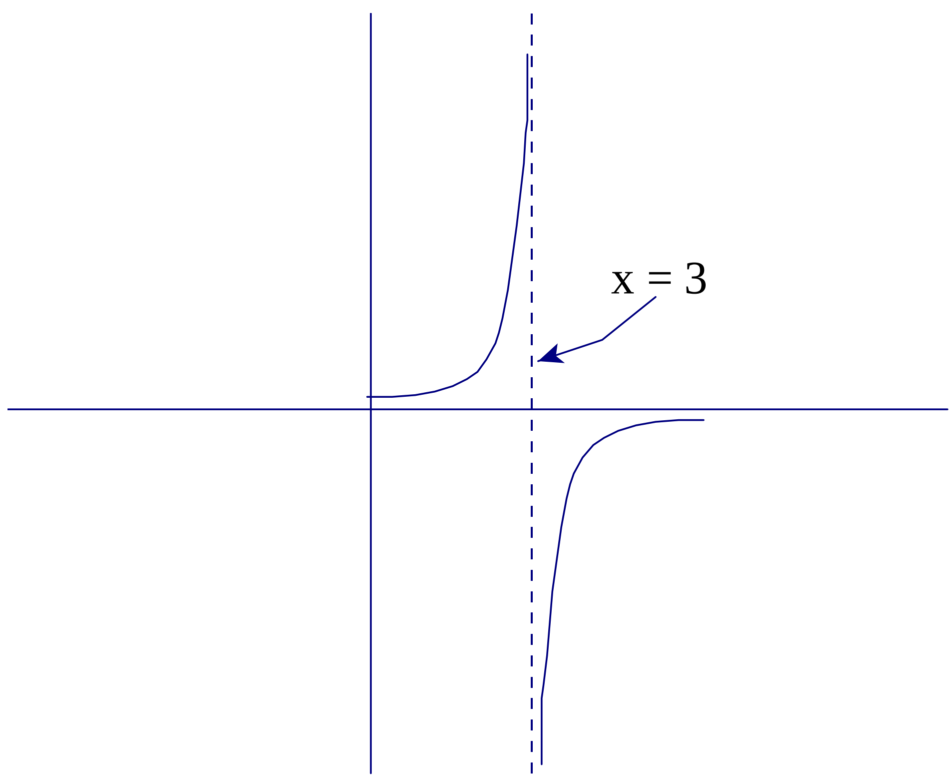
$x =$	$f(x) =$
3.5	-10.1
3.1	-100.8
3.01	$-1,000.3$
3.001	$-10,000.3$
3.0001	$-100,000.9$

Based on the information in the table above, do the following:

(a) $\lim_{x \to 3^-} f(x) = +\infty$

(b) $\lim_{x \to 3^+} f(x) = -\infty$

(c) Graph $f(x)$



8. Compute: $\lim_{x\to-\infty} \frac{9x^5+4x-8x}{3x^5-8x^2-5} =$

$\lim_{x\to-\infty} \frac{9x^5+4x-8x}{3x^5-8x^2-5} = \lim_{x\to-\infty} \frac{9x^5}{3x^5} = \lim_{x\to-\infty} (3) = 3$

$$\boxed{\text{i.e., } \lim_{x\to-\infty} \frac{9x^5+4x-8x}{3x^5-8x^2-5} = 3}$$

Extra (5 pts - WOW!)

Compute, using the properties of limits. Document each step.

$$\lim_{x\to1}\left[\left(3x^2-2x\right)\left(x^2-5x+3\right)\right] = \underbrace{\left[\lim_{x\to1}\left(3x^2-2x\right)\right]\left[\lim_{x\to1}\left(x^2-5x+3\right)\right]}_{\text{Limit of a product equals the product of the limits}} =$$

9. $$\underbrace{\left(\lim_{x\to1} 3x^2 - \lim_{x\to1} 2x\right)\left(\lim_{x\to1} x^2 - \lim_{x\to1} 5x + \lim_{x\to1} 3\right)}_{\text{Limit of a sum or difference equals the sum or difference of the limits}} =$$

$$\underbrace{\left(3\lim_{x\to1} x^2 - 2\lim_{x\to1} x\right)\left(\lim_{x\to1} x^2 - 5\lim_{x\to1} x + \lim_{x\to1} 3\right)}_{\text{Limit of a constant times a function equals the constant times the limit of the function}} =$$

$$\underbrace{\left(3\lim_{x\to1} x^2 - 2\lim_{x\to1} x\right)\left(\lim_{x\to1} x^2 - 5\lim_{x\to1} x + 3\right)}_{\text{The limit of a constant is the constant itself}} =$$

$$\underbrace{\left(3\lim_{x\to1} (1)^2 - 2\lim_{x\to1} (1)\right)\left(\lim_{x\to1} (1)^2 - 5\lim_{x\to1} (1) + 3\right)}_{\substack{\lim_{x\to c} x = c \\ \lim_{x\to c} x^n = c^n}} = -1$$