Proofs Involving Classic Theorems of Functions

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Exercises

1. **Prove:** If $f : A \longrightarrow B$ is one to one and $g : B \longrightarrow C$ is one to one, then $g \circ f : A \longrightarrow C$ is one to one. (i.e., The composition of one to one functions is one to one.)

2. Prove: If $f : A \longrightarrow B$ is onto and $g : B \longrightarrow C$ is onto, then $g \circ f : A \longrightarrow C$ is onto. (i.e., The composition of onto functions is onto.)

3. If $g \circ f : A \longrightarrow C$ is one to one, is either $f : A \longrightarrow B$ or $g : B \longrightarrow C$ one to one? Prove or Disprove. (Hint: when looking for a counter-example, strive for a simple counter-example; one where neither the domain nor the range has more than two or three elements.)

4. If $g \circ f : A \longrightarrow C$ is onto, is either $f : A \longrightarrow B$ or $g : B \longrightarrow C$ onto? Prove or Disprove. (Hint: when looking for a counter-example, strive for a simple counter-example; one where neither the domain nor the range has more than two or three elements.)

5. Prove: Function composition is associative. (i.e. $(h \circ g) \circ f = h \circ (g \circ f)$).

6. Prove: If $f : A \longrightarrow B$ and $g : B \longrightarrow C$ have inverses, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

7. Prove or disprove: A real valued function $f(x)$ is one to one if and only if it is monotone increasing or monotone decreasing.

8. Prove: A function $f : A \longrightarrow B$ has an inverse $f^{-1} : B \longrightarrow A$ if and only if f is one to one and onto.

9. Prove: $f : A \longrightarrow B$ is one to one and onto, if and only if $f^{-1} \circ f = 1_A$ and $f \circ f^{-1} = 1_B$.

10. Find a counter-example to show that $g \circ f = 1_A$ is not sufficient to guarantee that $f \circ g = 1_B$. (Hint: Strive for a simple counter-example - one where neither the domain nor the range has more than two or three elements.)