MTH 1125 - Test #1

FALL 2007

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Instructions. You may NOT use calculators.

Show CLEARLY how you arrive at your answers.

1. Compute: $\lim_{x\to 4}\frac{x^2-6x+8}{x^2-2x-8}=$

   1. Try Plugging in:

      $\lim_{x\to 4}\frac{x^2-6x+8}{x^2-2x-8}=\frac{(4)^2-6(4)+8}{(4)^2-2(4)-8}=\frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

      $\lim_{x\to 4}\frac{x^2-6x+8}{x^2-2x-8}=\lim_{x\to 4}\frac{(x-2)(x-4)}{(x+2)(x-4)}=\lim_{x\to 4}\frac{(x-2)}{(x+2)}=\frac{(4)-2}{(4)+2}=\frac{2}{6}=\frac{1}{3}$

      $\boxed{\text{i.e., } \lim_{x\to 4}\frac{x^2-6x+8}{x^2-2x-8}=\frac{1}{3}}$

2. Compute: $\lim_{x\to 1}\frac{x^2-4}{x^2-x-6}=$

   1. Try Plugging in:

      $\lim_{x\to 1}\frac{x^2-4}{x^2-x-6}=\frac{(1)^2-4}{(1)^2-(1)-6}=\frac{1}{2}$

      $\boxed{\text{i.e., } \lim_{x\to 1}\frac{x^2-4}{x^2-x-6}=\frac{1}{2}}$

3. Compute: $\lim_{x\to -2}\frac{\sqrt{11+x}-3}{x+2}=$

   (a) 1. Try Plugging in:

      $\lim_{x\to -2}\frac{\sqrt{11+x}-3}{x+2}=\frac{\sqrt{11+(-2)}-3}{(-2)+2}=\frac{0}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

      $\lim_{x\to -2}\frac{\sqrt{11+x}-3}{x+2}=\lim_{x\to -2}\frac{\sqrt{11+x}-3}{x+2}\cdot\frac{\sqrt{11+x}+3}{\sqrt{11+x}+3}=\lim_{x\to 2}\frac{\left(\sqrt{11+x}\right)^2-(3)^2}{(x+2)\left[\sqrt{11+x}+3\right]}=$

      $\lim_{x\to 2}\frac{(11+x)-9}{(x+2)\left[\sqrt{11+x}+3\right]}=\lim_{x\to 2}\frac{x+2}{(x+2)\left[\sqrt{11+x}+3\right]}=\lim_{x\to 2}\frac{1}{\left[\sqrt{11+x}+3\right]}=$

      $=\frac{1}{\left[\sqrt{11+(-2)}+3\right]}=\frac{1}{6}$

      $\boxed{\text{i.e., } \lim_{x\to -2}\frac{\sqrt{11+x}-3}{x+2}=\frac{1}{6}}$

4. Compute: $\lim_{x\to 2} \frac{x^2+2x+1}{x^2-5x+6} =$

   1. Try Plugging in:

      $\lim_{x\to 2} \frac{x^2+2x+1}{x^2-5x+6} = \frac{(2)^2+2(2)+1}{(2)^2-5(2)+6} = \frac{9}{0}$ No Good - Zero Divide!

   2. Try Factoring and Cancelling:

      No Good - Cancelling will only work when Step #1 yields $\frac{0}{0}$.

   3. Evaluate the one-sided limits:

      $\lim_{x\to 2^-} \frac{x^2+2x+1}{x^2-5x+6} = \lim_{x\to 2^-} \frac{x^2+2x+1}{(x-2)(x-3)} = \frac{9}{(-\varepsilon)(-1)} = \frac{9}{(\varepsilon)(1)} = \frac{(9)}{\varepsilon} = +\infty$

      $$\begin{array}{|ll|} \hline & x \to 2^- \\ \Rightarrow & x < 2 \\ \Rightarrow & x - 2 < 0 \\ \hline \end{array}$$

      $\lim_{x\to 2^+} \frac{x^2+2x+1}{x^2-5x+6} = \lim_{x\to 2^+} \frac{x^2+2x+1}{(x-2)(x-3)} = \frac{9}{(\varepsilon)(-1)} = \frac{9}{(\varepsilon)(-1)} = \frac{(9)}{-\varepsilon} = -\infty$

      $$\begin{array}{|ll|} \hline & x \to 2^+ \\ \Rightarrow & x > 2 \\ \Rightarrow & x - 2 > 0 \\ \hline \end{array}$$

      Since the one-sided limits are not equal, $\lim_{x\to 2} \frac{x^2+2x+1}{x^2-5x+6}$ Does Not Exist.

5. $f(x) = \frac{x+2}{3-x}$ Find the asymptotes and graph.

Verticals Look for those x-values that cause division by zero.

$\Rightarrow 3 - x = 0$

$\Rightarrow x = 3$ is a *possible* vertical asymptote.

Compute the one-sided limits of $f(x)$, as x approaches 3.

$\lim_{x \to 3^-} \frac{x+2}{3-x} = \frac{5}{\varepsilon} = +\infty$

$$\begin{array}{ll} & x \to 3^- \\ \Rightarrow & x < 3 \\ \Rightarrow & 0 < (3-x) \end{array}$$

↖ Infinite limits indicate

that $x = 3$ IS a

↙ vertical asymptote

$\lim_{x \to 3^+} \frac{x+2}{3-x} = \frac{5}{-\varepsilon} = -\infty$

$$\begin{array}{ll} & x \to 3^+ \\ \Rightarrow & x > 3 \\ \Rightarrow & 0 > (3-x) \end{array}$$

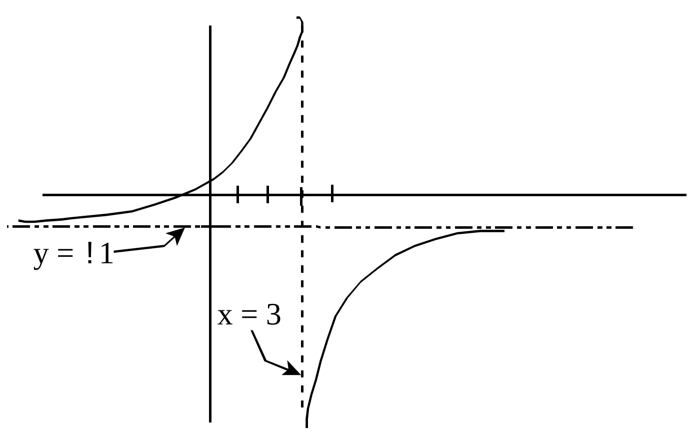
Horizontals Compute the limits as $x \to \pm\infty$

$\lim_{x \to -\infty} \frac{x+2}{3-x} = \lim_{x \to -\infty} \frac{x}{-x} = \lim_{x \to -\infty} (-1) = -1$

↖ Finite, constant limits indicate
that $y = -1$ IS a
↙ horizontal asymptote

$\lim_{x \to +\infty} \frac{x+2}{3-x} = \lim_{x \to +\infty} \frac{x}{-x} = \lim_{x \to +\infty} (-1) = -1$

Graph $f(x) = \frac{x+2}{3-x}$



6. ~

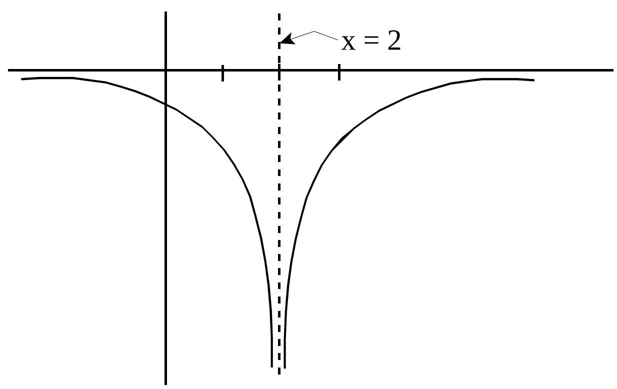
x	$f(x)$
1.0	-6.17
1.5	-88.36
1.9	-978.78
1.99	-9968.12
1.999	-124877.79

x	$f(x)$
3.0	-6.17
2.5	-88.36
2.1	-978.78
2.01	-9968.12
2.001	-124877.79

(a) $\lim_{x\to 2^-} f(x) = -\infty$ (as x approaches 2 through values less than 2, $f(x)$ gets unboundedly large in the negative direction.)

(b) $\lim_{x\to 2^+} f(x) = -\infty$ (as x approaches 2 through values greater than 2, $f(x)$ gets unboundedly large in the negative direction.)

(c) Sketch a graph of $f(x)$



7. Compute, using the properties of limits. Document each step.

$$\lim_{x\to 1}\left[\frac{3x^2-2x}{x^2-5x+3}\right] = \underbrace{\frac{\lim_{x\to 1}(3x^2-2x)}{\lim_{x\to 1}(x^2-5x+3)}}_{\text{The limit of a quotient equals the quotient of the limits}} = \underbrace{\frac{\lim_{x\to 1}3x^2-\lim_{x\to 1}2x}{\lim_{x\to 1}x^2-\lim_{x\to 1}5x+\lim_{x\to 1}3}}_{\text{The limit of a sum or difference equals the sum or difference of the limits}}$$

$$= \underbrace{\frac{3\lim_{x\to 1}x^2-2\lim_{x\to 1}x}{\lim_{x\to 1}x^2-5\lim_{x\to 1}x+\lim_{x\to 1}3}}_{\text{The limit of a constant times a function equals the constant times the limit of the function}} = \underbrace{\frac{3(1)^2-2(1)}{(1)^2-5(1)+\lim_{x\to 1}3}}_{\lim_{x\to c}x^n=c^n} = \underbrace{\frac{3(1)^2-2(1)}{(1)^2-5(1)+3}}_{\text{The limit of a constant is the constant itself}}$$

$$= \frac{1}{-1} = -1$$

$$\boxed{\text{i.e., } \lim_{x\to 1}\frac{(3x^2-2x)}{(x^2-5x+3)} = -1}$$